Thinning out Steiner trees: a node-based model for uniform edge costs

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Abstract. The Steiner Tree Problem is a challenging NP-hard problem. Many hard instances of this problem are publicly available, that are still unsolved by state-of-the-art branch-and-cut codes. A typical strategy to attack these instances is to enrich the polyhedral description of the problem, and/or to implement more and more sophisticated separation procedures and branching strategies. In this paper we investigate the opposite viewpoint, and try to make the solution method as simple as possible while working on the modeling side. Our working hypothesis is instead that the extreme hardness of some classes of instances mainly comes from over-modeling, and that some instances can become quite easy to solve when a simpler model is considered. In other words, we aim at "thinning out" the usual models for the sake of getting a more agile framework. In particular, we focus on a model that only involves node variables, which is rather appealing for the "uniform" cases where all edges have the same cost. We show that this model allows one to quickly produce very good (sometimes proven optimal) solutions for notoriously hard instances from the literature. In particular, we report improved solutions for several STEINlib instances, including the (in)famous hypercube ones.
Even though we do not claim our approach can work well in all cases, we report surprisingly good results for a number of unsolved instances. In some cases, our approach takes just few seconds to prove optimality for instances never solved (even after days of computation) by the standard methods.

1 Introduction

The Steiner Tree Problem (STP), in any of its various versions, is a challenging NP-hard problem that involves two related decisions: choosing the nodes to cover, and then covering them at minimum cost. Once the first decision has been taken, the second one is just trivial as it amounts to solving a minimum-cost tree spanning the selected nodes.

In this paper we introduce a new Mixed-Integer Linear Programming (MIP) approach for solving hard instances of the Steiner tree problem (close) to optimality. Instead of modeling graph connectivity using edge or arc variables (where many of them can exist), we propose to model it by using node variables only. Our model is particularly suited for "uniform" cases where all edges have the same cost. Besides the fact that these node-based models contain significantly less variables, they also avoid equivalences induced by uniform edge weights. For very dense graphs, or those containing a lot of symmetries, this strategy significantly outperforms the standard models where connectivity is modeled by using edge variables.

Our approach works for different variants of the Steiner tree problem, including the Prize-Collecting STP (PCSTP), the Node-Weighted STP (NWSTP) and also the Maximum-Weight Connected Subgraph Problem (MWCS). To have a unified framework, in the following we will therefore focus on a slightly more general variant of the PCSTP, with a (potentially empty) set of terminal nodes. This general problem definition covers both the classical Steiner tree problem in graphs and its Prize-Collecting counterpart, as special cases. Necessary adaptations for the remaining problems will be explained below.

Definition 1 (The prize-collecting Steiner tree problem (PCSTP)). *Given an undirected graph $G = (V, E)$ with a (possibly empty) set of* real terminals $T_r \subset V$*, edge costs $c : E \mapsto \mathbb{R}^+$ and node revenues $p : V \mapsto \mathbb{R}^+$, the PCSTP is to find a subtree $\mathcal{T}$ that spans all real terminals and such that the cost*

$$c(\mathcal{T}) = \sum_{e \in E[\mathcal{T}]} c_e + \sum_{i \notin V[\mathcal{T}]} p_i$$

is minimized.

In the classical PCSTP version studied in the previous literature, $T_r = \emptyset$, and the problem can be equivalently stated as searching for a subtree that maximizes the difference between the collected node revenues ($\sum_{i \in V[\mathcal{T}]} p_i$) and the costs for establishing the links of that tree ($\sum_{e \in E[\mathcal{T}]} c_e$). One objective value can be transformed into another by subtracting $c(\mathcal{T})$ from the sum of all node revenues ($P = \sum_{i \in V} p_i$).

In general, each node in $V \setminus T_r$ is considered as a *Steiner node*, i.e., it can be used as an intermediate node to connect real terminals, or those with positive revenues.

Observe that there always exists an optimal PCSTP solution in which nodes with zero revenue are not leaves. The same holds for each node $i \in V$ such that

$p_i > 0$ and $\min_{\{i,j\}\in E} c_{ij} > p_i$. (Note that we impose strict inequality in the latter condition.) Hence, only a specific subset of nodes in the PCSTP can be leaves of an optimal solution. We will refer to those nodes as *potential terminals*.

Definition 2 (Potential terminals). *Among the nodes $i \in V \setminus T_r$, only those with revenues $p_i > 0$ such that at least one adjacent edge is strictly cheaper than p_i are considered as potential leaves. These nodes are referred to as* potential terminals*, and the associated set is denoted by T_p:*

$$T_p = \{v \in V \setminus T_r \mid \exists \{u, v\} \text{ s.t. } c_{uv} < p_v\}.$$

In the following, we will call the set $T = T_r \cup T_p$, the set of *terminal nodes*. Our general problem definition covers the Steiner tree problem in graphs, since in this case all node revenues are equal to zero ($p_v = 0$, for all $v \in V$) and the set of real terminals is non empty, i.e., $\emptyset \neq T = T_r \subset V$.

2 A node-based MIP model

Node-based models for solving the Maximum-Weight Connected Subgraph Problem have been compared, both theoretically and computationally, in a recent publication [1]. Since there are no edge-costs involved in the objective of the MWCS, a natural MIP modeling approach is to derive a formulation in the space of node variables only. The first node-based model has been proposed in [3], and has been shown to computationally outperform extended formulations (involving both edge and node variables) on a benchmark set of instances from bioinformatics applications. However, as demonstrated in [1], the cycle-elimination model of [3] provides arbitrarily bad lower bounds and can be computationally improved by considering the notion of node-separators whose definition is provided below.

For STP/PCSTP instances, uniform edge costs can be embedded into node revenues (as shown below), so that using node-based MIPs appears natural in this case.

Definition 3 (Node Separators). *For two distinct nodes k and ℓ from V, a subset of nodes $N \subseteq V \setminus \{k, \ell\}$ is called (k, ℓ) node separator if and only if after eliminating N from V there is no (k, ℓ) path in G. A separator N is minimal if $N \setminus \{i\}$ is not a (k, ℓ) separator, for any $i \in N$. Let $\mathcal{N}(k, \ell)$ denote the family of all (k, ℓ) separators.*

Note that in order to make sure that a subset of chosen nodes is connected, it is sufficient to impose connectivity between the pairs of terminals (due to the minimization of the objective function). Therefore, we are mainly interested in node separators between pairs of terminals.

Let $N_\ell = \cup_{k\in T, k\neq\ell}\mathcal{N}(k, \ell)$ be the family of all such node separators with respect to a node $\ell \in T$. We will refer to elements from N_ℓ as ℓ-separators. Finally, let $\mathcal{N} = \cup_{\ell\in T}\mathcal{N}_\ell$ be the set of all node subsets that separate two arbitrary terminals.

Let us assume that we are dealing with an undirected graph G with node revenues p_v's and uniform edge costs $c_e = c$ for all $e \in E$.

In order to derive a node-based model, we will first shift edge costs into node costs as follows:

$$\tilde{c}_v = c - p_v, \quad \forall v \in V.$$

Let $P = \sum_{v \in V} p_v$ be the sum of all node revenues in G. Binary node-variables y_v, $\forall v \in V$, will be set to one if node v is part of the solution, and the node-based MIP model can be derived as follows:

$$(\text{PCSTP}_y) \qquad \min \sum_{v \in V} \tilde{c}_v y_v + (P - c) \tag{1}$$

$$y(N) \geq y_i + y_j - 1 \qquad \forall i, j \in T, i \neq j, \quad \forall N \in \mathcal{N}(i,j) \tag{2}$$

$$y_v = 1 \qquad \forall v \in T_r \tag{3}$$

$$y_v \in \{0, 1\} \qquad \forall v \in V \setminus T_r \tag{4}$$

where $y(N) = \sum_{v \in N} y_v$.

Connectivity constraints (2) are used to ensure connectivity of the underlying solution. Basically, whenever two distinct terminals $i, j \in T$ are part of a solution, at least one node from any node-separator $N \in \mathcal{N}(i,j)$ has to be chosen as well, in order to ensure that there exists a path between i and j.

There is a difference between our model and the one considered in [1], where a more general MWCS variant on digraphs has been studied. In this latter variant, a root node needs to be established, i.e., a node r with in-degree zero and such that there is a directed path from r to any node j being part of the solution. Consequently, an additional set of node variables was needed to locate the root, node separators were defined on digraphs, and connectivity constraints have been lifted with respect to root variables. Since our input graphs are undirected, we rely on the undirected model to keep the number of variables as small as possible.

Node-degree inequalities. The following node-degree inequalities are also valid for our model:

$$y(A_i) \geq \begin{cases} y_i, & \text{if } i \in T \\ 2y_i, & \text{otherwise} \end{cases} \tag{5}$$

where $A_i = \{v \in V \mid \exists \{v, i\} \in E\}$ is the set of all neighboring nodes of i. For terminals, these constraints ensure that at least one of their neighbors is part of the solution (assuming $\sum_{v \in V} y_v \geq 2$, which can be safely assumed after preprocessing single-node solutions). Clearly, they are just a special case of the inequalities (2), but can be used to initialize the model for a branch-and-cut approach. For the remaining nodes, constraints (5) make sure that each such node that belongs to a solution will be used as an intermediate node, i.e., at least two of its neighbors have to be included in the solution as well. These constraints are not implied by (2), in fact they can improve the quality of lower bounds of the original model.

It is sufficient to consider only minimal node-separators in inequalities (2) (since they dominate the remaining ones). In order to derive minimal node separators associated to node-degree constraints, we observe that nodes from $V \setminus T$ that are only adjacent to i and other nodes from A_i do not play a role in connecting i to the remaining terminals. Let J be the set of all such neighbors of a given $i \in T$, i.e., $J = \{j \in A_i \setminus T \mid A_j \subseteq A_i \cup \{i\}\}$. If $V \setminus A_i$ contains other terminals, then $A_i \setminus J$ is a minimal i-separator, i.e., $A_i \setminus J \in \mathcal{N}_i$ and we can correspondingly strengthen the node-degree inequalities (5).

Finally, observe that for potential terminals $i \in T_p$, the node-degree inequality can be lifted as follows:

$$2 \sum_{v \in A_i : c_{vi} < p_i} y_v + \sum_{v \in A_i : c_{vi} \geq p_i} y_v \geq 2y_i \quad \forall i \in T_p.$$

2-Cycle inequalities. Observe the following: if node $i \in V$ is adjacent to a node $j \in T_p$, so that $c_{ij} < p_j$, then if i is part of the optimal solution, j has to be included as well, i.e.,

$$y_i \leq y_j \quad i \in V, j \in T_p, c_{ij} < p_j \tag{6}$$

2.1 Separation of connectivity constraints

Whenever we want to cut off a fractional solution $\tilde{y}$, we can separate the connectivity cuts (2) by applying a maximum flow algorithm. For each pair of distinct terminals (i, j) such that $\tilde{y}_i + \tilde{y}_j - 1 > 0$, one would need to find a minimum (i, j)-cut in a support digraph D which is constructed as follows. First, each edge $e \in E$ is replaced by two directed arcs. Then, each node $v \in V \setminus \{i, j\}$ is replaced by an arc (v', v'') whose capacity is defined as $\tilde{y}_v$, all arcs entering v are now directed into v', and all arcs leaving v are now directed out of v''. Capacities of these arcs are set to ∞. Since all arcs except the node-arcs are of infinite capacity, the maximum (i, j)-flow returns the desired violated connectivity constraint.

According to our computational experience, however, the above procedure is rather time consuming (all terminal pairs need to be considered, and for each pair, the maximum flow is calculated). As there is always a certain trade-off between the quality of lower bounds obtained by separating fractional points and the associated computational effort, we refrain from the separation of fractional points in our default implementation.

Consequently, to ensure the validity of our branch-and-cut approach, we need to cut off infeasible integer points enumerated during the branching procedure (or detected by the heuristics of the MIP solver, given that the solver was not provided a complete information about the structure of the problem). Infeasible points are cut off by means of a `LazyCutCallback` in our setting based on the commercial MIP solver IBM ILOG CPLEX. For a given pair of distinct terminal nodes $i, j \in T$ such that $\tilde{y}_i = \tilde{y}_j = 1$, our separation procedure runs in linear time (with respect to $|E|$) and works as outlined below.

To derive our algorithm, we use the following well-known property of node-separators (see, e.g., [14]):

Lemma 1. *Let $N \in \mathcal{N}(i,j)$ be an (i,j) separator for $i, j \in T$, $i \neq j$, and let C_i and C_j be connected components of $G - N$ such that $i \in C_i$, $j \in C_j$. Then N is a minimal (i,j) separator iff every node in N is adjacent to at least one node in C_i and to at least one node in C_j.*

Let $\tilde{y}$ be an integer solution, and let $G_{\tilde{y}} = (V, E_{\tilde{y}})$ denote the "support graph" induced by $\tilde{y}$, where

$$E_{\tilde{y}} = \{\{i,j\} \in E \mid \tilde{y}_i = \tilde{y}_j = 1\}$$

If $\tilde{y}$ is infeasible, then $G_{\tilde{y}}$ contains at least two connected components, say C_i and C_j, with $i \in C_i$, $j \in C_j$, and $\tilde{y}_i = \tilde{y}_j = 1$. Let $A(C_i)$ be the set of neighboring nodes of C_i in G, i.e., $A(C_i) = \{v \in V \setminus C_i \mid \exists \{u, v\} \in E, u \in C_i\}$. Clearly, $\{i,j\} \notin E$ and hence $A(C_i) \in \mathcal{N}_i$. However, $A(C_i)$ is not necessarily a minimal (i,j) node separator, and Algorithm 1 below describes how to derive a minimal separator starting from $A(C_i)$.

Data: Infeasible solution defined by a vector $\tilde{y} \in \{0,1\}^n$ with $\tilde{y}_i = \tilde{y}_j = 1$, C_i being the connected components of $G_{\tilde{y}}$ containing i, and $j \notin C_i$.
Result: A minimal node separator N that violates inequality (2) with respect to i, j.
Delete all edges in $E[C_i \cup A(C_i)]$ from G
Find the set R_j of nodes that can be reached from j
Return $N = A(C_i) \cap R_j$

Algorithm 1: A linear time algorithm for detecting a minimal node separator between two components C_i and C_j in $G_{\tilde{y}}$.

In practice, set R_j can be found by just running a standard Breadth-First Search (BFS) on the original graph G, starting from node j, with the additional rule that nodes in $A(C_i)$ are never put in the BFS queue.

Proposition 1. *Algorithm 1 returns a minimal node separator $N \in \mathcal{N}(i,j)$ in time $O(|E|)$.*

Proof. By definition of N, i and j are not connected in $G - N$. To see that N is a minimal (i,j) separator, consider $G - N$ and let C'_i and C'_j be two connected components, containing i and j, respectively. Clearly, $C_i \subset C'_i$ and $C'_j = R_j \setminus N$. Hence, by Lemma 1, it follows that N is minimal. □

In case $p > 2$ connected components exist in $G_{\tilde{y}}$ (each of them containing at least one terminal), one can repeat the procedure described in Algorithm 1 for each pair of distinct components C_i and C_j.

We conclude this section by observing that, for the pure STP case with real terminals only, connectivity constraints translate into

$$y(N) \geq 1, \quad \forall N \in \mathcal{N}$$

where $\mathcal{N}$ is the family of all node separators between arbitrary real terminal pairs. Our model can therefore be interpreted as set covering problem with an exponential number of elements to be covered. As demonstrated by our computational experiments, this property can be exploited to derive specialized set covering heuristics for pure STP instances with uniform costs, with a significant performance boost.

3 Algorithmic framework

The proposed node-based model can be solved by means of a Branch-and-Cut (B&C) algorithm, to be initialized with a high-quality feasible solution and with a suitable set of relevant connectivity constraints.

Our initial MIP model, called the *basic model* in what follows, is made by (1), (3), (4), plus the node-degree inequalities (5) and the 2-cycle inequalities (6).

The overall algorithmic framework is shown in Algorithm 2. In an *initialization phase*, an initial solution pool $\mathcal{S}_{\text{init}}$ is generated by means of some problem-dependent heuristics. In a subsequent *local branching phase*, multiple calls of the B&C algorithm are used to explore the neighborhood of starting solutions chosen from the solution pool $\mathcal{S}_{\text{init}}$. All connectivity constraints separated during this phase are globally valid, hence they are stored in a global cut pool *CutPool*, and added to the initial MIP model before each B&C re-execution. The incumbent solution (denoted by *Sol*) is updated correspondingly.

The local branching phase implements a multiple restart policy (with different seed values), intended to gather relevant information about the problem at hand, namely good primal solutions and a relevant set of connectivity constraints; see e.g. [12] for a recent application of multiple restarts to MIPs with uncertainty. The availability of such information at the root node of each B&C re-execution turns out to be very important, as it triggers a more powerful preprocessing as well as the generation of a bunch of useful general-purpose cuts (in particular, $\{0, 1/2\}$-cuts [2, 4]) based on the problem formulation explicitly available on input.

Every ten local branching iterations, a recombination of the best solutions $\mathcal{S}$ is performed as follows. We create a subgraph induced by the union of all nodes presented in previously detected best solutions of each iteration. We first perform B&C on this subgraph (imposing a time limit and/or solution limit). If an improved solution is found, local branching continues from there. Otherwise, after $maxLBiter$ iterations, we start the final B&C (on the whole graph G) which is executed starting from the current *CutPool* and from the best found solution. The algorithm terminates after proving the optimality, or after reaching the given time limit.

3.1 Local branching

Local branching (LB) has been proposed in [11] as a solution approach that uses the power of a general-purpose MIP solver as a black box to strategically explore promising solution subspaces. LB is in the spirit of large-neighborhood

Data: Input graph G, instance of the STP/PCSTP/NWSTP/MWCS, iteration and time limits
Result: A (sub)-optimal solution *Sol*.
$\mathcal{S}_{\text{init}}$ =`InitializationHeuristics`()
$k = 1$, $CutPool = \emptyset$, $\mathcal{S} = \emptyset$
Choose *Sol* from the solution pool $\mathcal{S}_{\text{init}}$.
while *($k \leq maxLBiter$) and (time limit not exceeded)* **do**
  ($CutPool, Sol, \mathcal{S}$)= `LocalBranching`($CutPool, Sol, seed, \mathcal{S}$)
  $k = k + 1$
  Choose *Sol* from the solution pool $\mathcal{S}_{\text{init}}$. Change *seed*.
  if i mod $10 == 0$ **then**
    Sol_{recomb} = `Recombine`($\mathcal{S}$)
    if *cost*(Sol_{recomb}) < *cost*(*Sol*) **then**
      $Sol = Sol_{recomb}$
    end
  end
end
Sol = `BranchAndCut`($CutPool, Sol, TimeLim$)
return *Sol*

Algorithm 2: Proposed algorithmic framework.

search metaheuristics, with the main difference that the underlying local search black-box algorithm is a MIP with a specific local branching constraint that restricts the search for an optimal solution within a certain neighborhood of a given reference solution.

The LB framework is built on top of our B&C solver. As already mentioned, our solver deals with two sets of inequalities: those in the *basic model*, that are always part of the model, and connectivity constraints (2) that are dynamically separated and stored in a global *CutPool* to be used in every subsequent B&C call.

Given a reference solution *Sol*, let $W_1 = \{v \in V \mid v \in Sol\}$ and $W_0 = V \backslash W_1$. The *symmetric* local branching constraint makes sure that the new solution is within radius r (say) from the solution *Sol* with respect to the Hamming distance between the two solutions, i.e.

$$\sum_{v \in W_0} y_v + \sum_{v \in W_1} (1 - y_v) \leq r$$

Alternatively, one may consider *asymmetric* local branching constraint, requiring that the new solution contains at least $|Sol| - r$ nodes from *Sol*:

$$\sum_{v \in W_1} (1 - y_v) \leq r. \tag{7}$$

Notice that, for a fixed radius, the neighborhood of the asymmetric version is larger and leads to potentially better solutions—though it is more time consuming to explore. For example, for $r = 0$ the asymmetric version is equivalent to fixing to 1 all the variables from *Sol*, so that many feasible solutions are still available even in the 0-neighborhood around *Sol*. After some preliminary tests, we decided to use the asymmetric LB constraint (7) in our implementation, with a small radius r ranging from 10 to 20. Working with a small radius is indeed crucial for the success of proximity methods such as LB, as recently pointed out in [13].

Since the goal of the B&C in this context is to quickly find high-quality solutions, we do not necessarily search for an optimal solution within the given neighborhood, but we rather impose limits on the number of incumbent solutions found (*SolLim*), and a (deterministic) time limit per iteration (*TimeLim*).

The neighborhood is systematically explored by starting with the initial radius r_{min}, and increasing it by r_{delta} each time the subproblem could not provide an improved solution. Each time an improved solution is found, the neighborhood radius is reset to by r_{min} The whole framework is executed until a given number of iterations (*MaxIter*) is reached, or the radius exceeds r_{max}. Note that the radius limit r_{max}, in combination with a consistent increase of the current radius, implicitly imposes a limit on the overall number of iterations without any improvement.

Data: Starting solution *Sol*, *CutPool*, $\mathcal{S}$, *seed*, lower and upper bound for radius (r_{min}, r_{max}), radius step (r_{delta}), maximum n. of iterations $MaxIter$.
Result: Improved solution *Sol*, enlarged cut pool *CutPool*.
$r = r_{min}$, $k = 1$
while $(k \leq MaxIter)$ *and* $(r \leq r_{max})$ **do**
  Add local branching constraint (7) centered on *Sol* with radius r
  $(Sol', CutPool')$ = `BranchAndCut`$(CutPool, Sol, TimeLim, SolLim)$
  Remove the local branching constraint from the current model
  if $cost(Sol') < cost(Sol)$ **then**
    $Sol = Sol'$, $r = r_{min}$
    $\mathcal{S} = \mathcal{S} \cup Sol$
  else
    $r = r + r_{delta}$
  end
  $CutPool = CutPool \cup CutPool'$
  $k = k + 1$
end
return *(CutPool, Sol,* $\mathcal{S}$*)*

Algorithm 3: Basic Local Branching.

3.2 Benders-like (Set Covering) Heuristic

Local branching has a primal nature, in the sense that it produces a sequence of feasible solutions of improved cost. In addition, it needs a starting feasible solution, that in some cases can be time consuming to construct. As a matter of fact, for some very large/hard classes of instances we found that a dual approach is preferable, that produces a sequence of infeasible (typically, disconnected) solutions and tries to repair them to enforce feasibility. Algorithm 4 illustrates a general dual scheme that can be viewed as a heuristic version of the well-known Benders' exact solution approach to general MIPs.

Data: Time/iteration limits.
Result: Feasible solution Sol, cut pool $CutPool$.
Sol = dummy solution of very large cost
$CutPool = \emptyset$
while *(time/iteration limit not exceeded)* **do**
  Heuristically solve a relaxation of the current model (including all cuts in $CutPool$) and let Sol^R be the possible disconnected solution found
  Add the local branching constraint (7) centered on Sol^R to the unrelaxed model
  $(Sol', CutPool') = \texttt{BranchAndCut}(CutPool, Sol, TimeLim, SolLim)$
  Remove the local branching constraint from the unrelaxed model
  if $cost(Sol') < cost(Sol)$ **then**
    $Sol = Sol'$
  end
  $CutPool = CutPool \cup CutPool'$
end
return Sol

Algorithm 4: A conceptual Benders-like heuristic.

In our experiments, we found that the above approach works very well for uniform STP instances of very large size, for which the standard MIP approach seems not very appropriate as even the LP relaxation of the model takes an exceedingly large computing time to be solved. Our *set-covering based heuristic* is an implementation of Algorithm 4 for these hard instances, and is based on their set covering interpretation. Indeed, as already observed, the basic model turns out to be a compact set covering problem where columns correspond to Steiner nodes, rows to real terminals not adjacent to any other real terminal, and column j covers row i iff $\{i, j\} \in E$. The approach can be outlined as follows.

At each iteration of the while loop, the relaxation to be heuristically solved is constructed through a procedure that automatically extracts a set covering relaxation from the current model. This is done by simply (1) projecting all fixed variables (including y variables for hard terminals) out of the model, and (2) skipping all constraints, if any, that are not of type $y(S) \geq 1$ for some node set S.

We then proceed by heuristically solving the set covering relaxation through an implementation of the Caprara-Fischetti-Toth (CFT) heuristic [5, 6]. This is a very efficient set covering heuristic based on Lagrangian relaxation, that is specifically designed for hard/large instances.

Given a hopefully good set covering solution Sol^R, we *repair* it in a very aggressive way by introducing a local branching constraint in asymmetric form with radius $r = 0$, and then by applying our B&C solver (with its run-time connectivity cut separation) with a short time/node limit. As already observed, this local branching constraint in fact corresponds to fixing $y_j = 1$ for all j such that $Sol^R_j = 1$. As a result, the size/difficulty of the MIP model after fixing is greatly reduced, hence the node throughput of B&C solver becomes acceptable even for large instances—while setting a larger radius would not result in a comparable speedup.

All violated connectivity cuts generated by the B&C are added to *CutPool* and hence to the current model. This makes solution Sol^R (if disconnected) infeasible even for the next set covering relaxation and thus the procedure can be iterated until an overall time/iteration limit is reached.

To improve diversification, our implementation uses the following two mechanisms:

– the procedure that extracts the set covering model makes a random shuffle of the rows/columns, so as to affect in a random way the performance of the CFT heuristic;
– before the repairing phase, we randomly skip (with a uniform probability of 20%) some variable fixings, meaning that approximately 80% of the variables y_j's with $Sol^R_j = 1$ are actually fixed.

As such, the performance of our final heuristic (though deterministic) is affected by the initial random seed, a property that can be very useful to produce different solutions in a multi-start scheme.

Finally, we observe that our current CFT implementation is sequential and cannot exploit multiple processors. We therefore decided to also run the refining B&C in single-thread mode, thus obtaining an overall sequential code that can be run in parallel and with different random seeds on each single core (in the multi-thread mode). The best found solution is finally returned.

This Benders-like heuristic is embedded in the overall algorithmic framework shown in Algorithm 2 as `InitializationHeuristics` for uniform STP instances on bipartite graphs with a large percentage of terminals. Indeed, these kinds of graph are very regular and the basic model likely gives a reasonable approximation of the STP problem—in the sense that most connectivity constraints are automatically satisfied. In addition, the while-loop in Algorithm 2 that would apply standard local branching after the set-covering based heuristic is unlikely to be effective for these graphs, so we set $maxLBiter = 0$ and skip it in this case.

We conclude this section by observing that the "relaxation" to be heuristically solved in the Benders-like scheme is not intended to produce valid dual bounds, as its purpose is to feed the refining procedure with good (possibly disconnected) solutions. As a matter of fact, one can think of the relaxation as a "blurred" version of the original problem, which retains some of its main features but is not bothered by too many details (namely, connectivity conditions) that would overload the model. Following this general idea, we implemented the following variant of our set-covering based heuristic, which is intended for non-uniform instances on bipartite graphs with a large percentage of terminals. The idea is that we heuristically move edge-cost information into node revenues, so as to get a "blurred" uniform instance on the same underlying graph. To be specific, for each node v we just subtract the average cost of its adjacent edges from the revenue p_v, and in the end we reset all edge costs to $c = 0$. Needless to say, these modified costs/revenues are only used within the CFT heuristic, while the original ones are used in the refining phase.

4 Computational Results

Our algorithms are applied to the following problems from the DIMACS challenge: STP, (rooted) PCSTP, MWCS, and also degree-constrained STP (DC-STP). We next summarize the results we obtained on the set of hard (unsolved) cases of the SteinLib [15] instance library. Additionally, we consider non-trivial MWCS instances posted at the website of the DIMACS challenge.

Detailed computational results, covering all nontrivial instances from the challenge are provided in the Appendix. Among these nontrivial instances, we distinguish between *easy* and *difficult* ones. The following criteria are applied: we first run all the instances using the exact B&C approach (without sophisticated heuristics and local branching) with a time-limit of one hour (with a fixed seed value) and record the obtained results. If an instance remains unsolved, it is considered as *difficult*. Our heuristic framework (consisting of the initialization and local branching phase, see Algorithm 2) is then applied to all *difficult* instances with a time limit of one hour (10 independent runs with different seeds).

The experiments were performed on a cluster of computers, each consisting of 20 cores (2.3 GHz) and with 64GB RAM available for 20 cores. Reported computing times are in wall-clock seconds. To limit the overall time needed to complete our experiments, we decided to allow up to five simultaneous 4-core runs on the same computer, which however implies a significant slowdown due to shared memory. CPLEX 12.6 was used as MIP-solver with an imposed memory limit of 16GB RAM.

For solving uniform instances, we used the proposed node-based model (which will be referred to as y-model in the following). For solving non-uniform instances, we used the Steiner arborescence model studied in [19] and referred to as (x, y)-model in the following. In all cases, the models are incorporated into the algorithmic framework shown in Algorithm 2.

4.1 Initial filter

Our implementation also contains a *filter* that analyzes the graph structure and the costs of the input graph. According to the graph properties, the algorithm then decides the actual MIP model, as well as the kind of initialization heuristics and the preprocessing, to apply.

The `InitializationHeuristics` works as follows: For planar and sparse graphs it consists of a dual ascent primal-dual approach followed by a parallel implementation of the partitioning heuristic (see [18]). For bipartite graphs, it is a Benders-like set cover heuristic described in Section 3.2 (non-uniform instances are transformed into uniform ones through the "blurring" scheme outlined at the end of Subsection 3.2).

In Table 1, rules to select the chosen model and its parameters are described. The y-model is the primary choice for uniform instances, but in the case of sparse instances, it is replaced with the (x, y)-model if the number of terminals is below a certain threshold.

Table 2 lists the rules that are applied to decide which initialization heuristics to choose. In the case of bipartite graphs, the CFT heuristic is applied for both uniform and non-uniform instances to generate high-quality starting solutions. For non-uniform instance, each node v is assigned the weight $\frac{1}{|\delta(v)|} \sum_{e \in \delta(v)} c_e$ ("blurred" CFT).

Furthermore, sparse STP instances are initialized with a set of connectivity cuts generated by dual ascent, and with a starting solution generated by a partitioning-based heuristic. Dense STP instances are instead preprocessed by using the Special Distance Test [10].

Table 1. Filter rules for choosing the solution algorithms based on instance structure. An instance is labeled as `sparse` if $|E|/|V| < 3$, as `dense` otherwise. The metric `ratioT` is defined by $|T|/|V|$. An instance is labeled as `bipartite` if the instance graph is bipartite with respect to the node sets $V \setminus T$ and T. The attribute `weightRange` is defined as the difference between the minimum and maximum edge weight.

Rule		Applied algorithm
`sparse` $\wedge$ `uniform` $\wedge$ `ratioT` > 0.4	$\rightarrow$	y-model
`sparse` $\wedge$ `uniform` $\wedge$ `ratioT` ≤ 0.4	$\rightarrow$	(x, y)-model, tailing-off=0
`sparse` $\wedge$ $\neg$`uniform`	$\rightarrow$	(x, y)-model, tailing-off=0
`dense` $\wedge$ `uniform`	$\rightarrow$	y-model
`dense` $\wedge$ $\neg$`uniform`	$\rightarrow$	(x, y)-model, tailing-off=0.001
`weightRange` < 10	$\rightarrow$	Allow non-improving moves during local search (cf. Section 4.2)

4.2 Implementation details of the (x, y)-model

In addition to node variables y defined above, this model uses arc variables x that model the rooted arborescence structure of the solution. For the STP, a random terminal is chosen as a root, and for the unrooted PCSTP an artificial root is introduced and connected to all terminals (with an additional constraint that out-degree of the root is one). This model has been studied in [19], and in this section we explain the the main differences between our implementation and the one given in [19].

Table 2. Filter rules for choosing initialization heuristics based on the instance structure and selected algorithm. STP means that the problem instance is a Steiner tree problem. XY denotes that the (x, y)-model is applied.

Rule							Initialization Heuristic
`dense`	$\wedge$	`uniform`	$\wedge$	`bipartite`		$\rightarrow$	CFT
`dense`	$\wedge$	$\neg$`uniform`	$\wedge$	`bipartite`		$\rightarrow$	"Blurred" CFT
`dense`	$\wedge$	`uniform`	$\wedge$	$\neg$`bipartite`		$\rightarrow$	Local Branching
`dense`	$\wedge$	$\neg$`uniform`	$\wedge$	$\neg$`bipartite`		$\rightarrow$	Local Branching
`dense`	$\wedge$	STP				$\rightarrow$	Preprocessing
`sparse`	$\wedge$	STP	$\wedge$	XY		$\rightarrow$	Dual Ascent
`sparse`	$\wedge$	STP				$\rightarrow$	Partitioning heuristic (cf. Section 4.2)

Initialization Heuristic: A pool of initial feasible solutions is constructed as follows. Several terminals are chosen as root nodes, for each of which a solution is calculated by applying the shortest path Steiner tree heuristic (see, e.g., [9]). In the PCSTP case, for a small number of iterations the set $T_p \cup T_r$ is perturbed, and subsets of terminals of different size are considered as fixed terminals T'. Then for each chosen set T', the same construction heuristic as for the STP is applied. Each solution is also improved through a local search (see below).

For sparse, (almost) planar non-uniform instances of the STP, our framework computes an additional, enhanced initial solution by applying a parallel variant of the partitioning heuristic described in [18]. Based on a randomly chosen solution from the pool, the input graph is partitioned into a set of smaller subgraphs (containing terminals and their closest Steiner nodes). The STP is then solved to optimality (or with a small time limit) on each of these subgraphs independently. The obtained disconnected (and thus, infeasible) solution is then repaired by a shortest-path like heuristic, followed by a local search. When run in a multi-thread mode, the STP solution on each subgraphs is assigned to a single thread.

Primal Heuristic: We apply a few rounds of rounding on the set of y variables, with different threshold values. Rounded up variables again defines set T' on which we run the shortest path Steiner tree heuristic (with a modified cost function that reflects the LP-values at the current branch-and-bound node) and prune Steiner leaves.

Local Search Heuristics: Solutions obtained by the initialization or the primal heuristic are further improved by applying several local search procedures: Key-Path-Exchange, (Key-)Node-Removal and (Key-)Node-Insertion. For further implementation details about these heuristics, see [21]. We only accept strictly improving moves, except for the uniform (or almost uniform) instances, for which (due to existence of many symmetric solutions) all moves with non-decreasing objective values are performed.

Additional Valid Inequalities: In [19] the authors observe that the (x, y)-model for PCSTP contains a lot of symmetries, and propose to get rid of them by fixing the terminal node with the smallest index (among those taken in the solution) to be the root node. This is imposed by adding additional asymmetry constraints. The latter constraints added by [19] were given in a disaggregated form, whereas in our implementation we add their stronger, aggregated variant: $\sum_{i>j} x_{ri} \leq 1 - y_j$, for all $j \in T$. We also add 2-cycle inequalities (6) to our starting model.

Separation Algorithms: For fractional solutions, connectivity constraints are separated by the calculation of the maximum flow (implemented through `UserCutCallback` of CPLEX). In order to find more diversified cuts as early as possible, we first add a single violated cut between the root and a single terminal, then we update the capacities on the arcs involved in this cut-set, and then iterate until all terminals are separated. Among the two cut-sets returned by the maximum flow algorithm of [8], we choose the one closer to the terminal (cut-sets closer to the root typically involve similar edges, and hence may imply many redundant cuts). For sparse or randomly generated non-uniform instances, the (x, y)-model provides very strong lower bounds, so that in most cases branching is not needed. On the other hand, for more difficult instances with large LP-gaps, *tailing off* plays a crucial role in the performance of the B&C algorithm. Rather than staying too long at the root node of the B&C tree (and separating too many cuts), we resort to branching if the lower bound does not improve much. Similarly, for the PCSTP, we only separate cuts between the root and those terminals i such that $y_i \geq 0.5$ in the fractional solution to separate.

Infeasible integer solutions are separated by searching for connected components in the support graph. For each subset S inducing a connected component of an infeasible solution, a generalized subtour elimination constraint $x(A(S)) \leq y(S) - y_i$ is added to the model, for $i \in S$ with the highest revenue.

4.3 Results for uniform STP instances

PUC dataset: A preliminary study of hypercube (hc) instances [20] confirmed their very high symmetry even in the space of node variables. Indeed, after removing the y variables of terminal nodes (that are all fixed to one), the symmetry group produces a single orbit that contains all the remaining variables. This implies that one can w.l.o.g. fix $y_v = 1$ for an arbitrary Steiner node $v \in V \setminus T$, thus significantly reducing the symmetry group—that however remains nontrivial.

It is well known that symmetry plays an prominent (negative) role when solving an instance to optimality through enumerative methods. However, symmetry can even be of help when heuristics come into play, as the impact of bad heuristic choices is mitigated somehow by symmetry. As our main goal was to provide improved heuristic solutions for the unsolved cases, we did not investigate symmetry properties any further, and actually preferred not to apply the symmetry-breaking fixing $y_v = 1$ mentioned above.

Instead, we concentrated on the set-covering heuristic described in Section 3.2. Table 3 reports the very preliminary results we obtained by an over-simplified version of this approach, where the CFT set-covering heuristic was only available to us as a binary executable—namely, the TURNI commercial implementation [16, 17]. As a result, this piece of software could not be fully embedded into our code and we had to use it off-line through input/output text files. In addition, we could not iterate the approach, i.e., we only refined a single set-covering solution. TURNI was therefore run only once and with a short time limit—one minute was enough for finding very good solutions for all cases, except for hc11u for which we allowed for 30 minutes. To our pleasant surprise, even in this very simplified setting the repairing phase turned out to be very fast and effective in producing connected solutions whose cost was very close to the cost of the set covering solution.

Table 3. Our very preliminary results for unsolved uniform STP instances of the PUC class

name	PreviousBest	NewBest	TURNI time (s.)	Repair time (s.)
bip52u	234	234	60.0	2.14
bip62u	220	**219**	60.0	0.03
bipa2u	341	**338**	60.0	0.05
hc9u	292	292	60.00	1.53
hc10u	581	**575**	60.00	4.52
hc11u	1154	**1145**	1800.00	0.07
hc12u	2275	**2267**	60.00	6.68

Significantly better results were obtained at a later stage of our experiments, when we implemented the CFT heuristic ourselves and were able to fully embed it into the Benders-like scheme of Algorithm 4; see Table 6.

GAPS and SP datasets: For the subgroup "skutella" (s3-s5) of the artificially generated uniform instance set GAPS for the STP, LP-gaps of the standard connectivity-based (x, y)-model are large. Standard MIP approaches for these instances have difficulties in closing the integrality gap. Table 4 reports our results obtained on instances s1 to s5 from GAPS, and clearly demonstrates the power of our y-model. Additionally, for two previously unsolved instances from the set SP (with uniform edge costs as well), namely w13c29 and w23c23, we provide optimal values. For these two latter instances, both models were able to prove the optimality, with significant speed-ups achieved by the y-model. It is worth mentioning that larger instances given in Table 4 are very erratic, with running times ranging between a few seconds, up to a few hundreds of seconds, depending on the seed value. Results given in Table 4 are produced by a single run, with a default seed value of 0.

Table 4. Uniform STP instances (GAPS and SP datasets). Proven optimal solutions in boldface. Previous best known solutions given in brackets. Column Time gives the computing time for proving the optimality (or, the time limit, otherwise). Columns UB and LB show upper and lower bounds obtained by the (x, y)-model, within the time limit of two hours, respectively.

				y-model		(x,y)-model			
name	$\|V\|$	$\|E\|$	$\|T\|$	OPT	Time (s.)	UB	LB	gap	Time (s.)
s1	64	192	32	**10**	0.03	10	10	0.0%	0.01
s2	106	399	50	**73**	0.10	73	73	0.0%	1.67
s3	743	2947	344	**514**	0.29	514	508	1.19%	7200.00
s4	5202	20783	2402	**3601**	1.72	3601	3515	2.39%	7200.00
s5	36415	145635	16808	**25210**	30.06	25210	24448	3.02%	7200.00
w13c29	783	2262	406	**507** (508)	0.61	507	507	0.00%	167.17
w23c23	1081	3174	552	**689** (694)	196.19	689	689	0.00%	1114.00

4.4 Results for MWCS instances

The MWCS can be transformed into the PCSTP with uniform edge costs (see [1]). We have tested both y- and (x, y)-model on the MWCS dataset, and the obtained computational results (for the most challenging instances) are reported in Table 5. We report best out of five runs (with different seed values).

Table 5. ACTMOD PCSTP instances from the DIMACS website.

					y-model		(x,y)-model	
name	$\|V\|$	$\|E\|$	$\|T\|$	Opt	Time (s.)	Gap	Time (s.)	Gap
drosa001	5226	93394	5226	8273.98263	17.1	**0.00**	56.0	**0.00**
drosa005	5226	93394	5226	8121.313578	53.8	**0.00**	1623.9	**0.00**
dros0075	5226	93394	5226	8039.859460	18.6	**0.00**	81.4	**0.00**
HCMV	3863	29293	3863	7371.536373	1.8	**0.00**	7.4	**0.00**
lymph	2034	7756	2034	3341.890237	0.2	**0.00**	0.8	**0.00**
mice_1	3523	4345	3523	11346.927189	3758.1	**0.00**	1.2	**0.00**
mice_2	3514	4332	3514	16250.235191	4.5	**0.00**	0.8	**0.00**
mice_3	2853	3335	2853	16919.620407	19.3	**0.00**	0.7	**0.00**

4.5 Summary of improved results for unsolved STP instances

Table 6 shows a summary of improved results obtained by our computational framework on the most difficult sets of unsolved instances from SteinLib (PUC and I640).

Table 6. Improved values for unsolved STP instances within one hour time limit. Filtering applied as described above. Column Diff shows the improvement with respect to the previous best known values published on [7] (*) Optimal solution found by B&C in a single run.

				BEST		AVG		STD		
Instance	$\|V\|$	$\|E\|$	$\|T\|$	UB	Time	UB	Time	UB	Time	Diff
bip52u	2200	7997	200	233	1368.55	233.80	277.51	0.42	575.62	-1
bip62u	1200	10002	200	219	6.30	219.00	12.12	0.00	4.83	-1
bipa2p	3300	18073	300	35355	540.54	35360.90	1303.05	4.38	867.45	-24
bipa2u	3300	18073	300	337	177.88	337.00	305.12	0.00	211.13	-4
hc10p	1024	5120	512	59981	256.73	60035.75	1324.04	34.71	918.08	-513
hc10u	1024	5120	512	575	11.33	575.00	115.98	0.00	143.77	-6
hc11p	2048	11264	1024	119404	3502.65	119532.00	2007.50	61.17	1279.12	-375
hc11u	2048	11264	1024	1145	666.33	1145.50	1481.97	0.53	1020.23	-9
hc12p	4096	24576	2048	236267	2764.17	236341.88	2309.54	61.04	656.21	-682
hc12u	4096	24576	2048	2261	2833.56	2262.89	2497.36	1.45	493.16	-14
cc10-2p	1024	5120	135	35298	289.17	35393.20	636.55	82.32	984.51	-81
cc11-2p	2048	11263	244	63635	1227.09	64007.40	1200.23	189.87	726.29	-191
cc3-10p	1000	13500	50	12782	3377.96	12820.30	2143.62	38.52	1377.73	-78
cc3-11p	1331	19965	61	15594	1383.32	15638.30	1682.92	35.32	1242.23	-15
cc3-12p	1728	28512	74	18837	2322.49	18905.50	1624.50	43.36	1182.90	-1
cc3-12u	1728	28512	74	185	29.07	185.00	150.37	0.00	163.39	-1
cc6-3p	729	4368	76	20293	3292.58	20367.70	2153.67	40.45	894.91	-163
cc6-3u*	729	4368	76	197	788.53					
cc7-3u	2187	15308	222	550	1148.94	554.20	1346.71	2.35	972.56	-2
cc9-2p	512	2304	64	17202	891.76	17291.10	1052.64	40.31	1101.80	-94
i640-312	640	4135	160	35770	455.41	35780.90	1899.64	26.38	1153.81	-1
i640-314	640	4135	160	35532	2981.68	35545.70	1623.32	7.78	973.20	-6
i640-315	640	4135	160	35720	634.72	35755.00	1115.30	14.28	928.91	-21

Note that the table reports the results obtained within the time limit of one hour only. By extending the time limit, or by using more than four threads in parallel, the obtained values can further be improved. For example, for the most difficult ones we obtain:

problem	instance	best UB	Time	#threads
STP	hc11u	**1144**	474	8
STP	hc12u	**2256**	4817	8
STP	hc12p	**236158**	4411	4
PCSTP	hc11u2	**751**	298	8
PCSTP	hc12u2	**1492**	632	16

5 Conclusions

We have presented a simple model for the Steiner tree problem, involving only node variables. Besides drastically reducing the number of the required variables, the removal of edge variables avoids a number of issues related to equivalent (possibly symmetrical) trees spanning a same nodeset. In this view, we are "thinning out" the usual edge-based model with the aim of getting a more agile framework. Our model is mainly intended to instances with uniform edge costs, but it can be extended to the general case by a simple edge splitting mechanism (possibly to be applied on the fly). Computational results show that our approach can dramatically improve the performance of an exact solver, and in some cases converts very hard problems into trivial-to-solve ones.

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Appendix: Detailed Computational Results

All experiments have been performed on a cluster of computers with similar performance, where a typical PC has 4 cores, 16GB RAM and runs at 2.3 GHz. CPLEX 12.6 has been used as MIP-solver with an imposed memory limit of 12GB RAM. A run of the DIMACS benchmark program measured 360.070111 trees/second.

6 Detailed Exact results

Difficult instances have been identified by running B&C with one hour timelimit without any complex heuristics. For the STP starting solutions have been produced through local search and through the partitioning heuristic in the case of sparse graphs (Vienna, ES1000FST, ES10000FST, Copenhagen, VLSI).

Exact results for the STP

Table 7. PUCN STP instances. Results computed by applying B&C to the y-model (time limit = 1 hr., CPLEX memory limit = 12GB).

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
cc10-2n	1024	5120	135	176	183	4.36	445926	42.34	memout
cc11-2n	2048	11263	244	317	331	4.42	562845	204.59	memout
cc12-2n	4096	24574	473	600	626	4.33	295685	914.77	memout
cc3-10n	1000	13500	50	68	75	11.39	280697	0.43	memout
cc3-11n	1331	19965	61	82	92	12.65	268909	1.76	memout
cc3-12n	1728	28512	74	99	111	12.50	313396	8.68	timeout
cc3-4n	64	288	8	13	13	0.00	34	0.08	0.32
cc3-5n	125	750	13	20	20	0.00	2965	0.05	0.53
cc5-3n	243	1215	27	42	42	0.00	140731	9.79	39.46
cc6-2n	64	192	12	18	18	0.00	16	0.01	0.15
cc6-3n	729	4368	76	100	100	0.00	40194	48.00	48.25
cc7-3n	2187	15308	222	280	294	5.32	442673	133.05	memout
cc9-2n	512	2304	64	94	100	6.82	591930	5.62	memout

Table 8. Copenhagen STP instances. Results computed by exact B&C on the (x, y)-model (timelimit=1hr).

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
ind1	18	31	10	604	604	0.00	0	0.08	0.09
ind2	31	57	10	9500	9500	0.00	0	0.07	0.08
ind3	16	23	10	600	600	0.00	0	0.04	0.06
ind4	74	146	25	1086	1086	0.00	0	0.08	0.09
ind5	114	228	33	1341	1341	0.00	0	0.14	0.15
rc01	21	35	10	25980	25980	0.00	0	0.01	0.04
rc02	87	176	30	41350	41350	0.00	0	0.08	0.08
rc03	109	202	50	54160	54160	0.00	0	0.16	0.16
rc04	121	197	70	59070	59070	0.00	0	0.11	0.14
rc05	247	486	100	74070	74070	0.00	0	0.19	0.20
rc06	2502	6244	100	79714	79714	0.00	7	17.52	64.40
rc07	2740	6578	200	108740	108740	0.00	9	70.20	85.50
rc08	7527	18170	200	112564	112564	0.00	7	2305.66	2307.83
rc09	6128	15264	200	111005	111005	0.00	0	1184.74	1184.81
rc10	1572	3245	500	164150	164150	0.00	0	13.37	13.38
rc11	2858	5819	1000	230837	230837	0.00	12	148.22	149.48
rt01	262	740	10	2146	2146	0.00	0	0.28	0.41
rt02	788	1938	50	45852	45852	0.00	0	4.10	4.10
rt03	1725	4092	100	7964	7964	0.00	0	29.84	30.31
rt04	9469	22743	100	9622	9716	0.98	0	3003.17	timeout
rt05	15473	38928	200	50280	51945	3.31	0	81.23	timeout

Table 9. PUC STP instances. Results computed by B&C using the (x, y)-model (time limit = 1 hr., CPLEX memory limit = 12GB)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
bip42p	1200	3982	200	24657	24657	0.00	136198	565.27	2150.50
bip42u	1200	3982	200	236	236	0.00	1997930	48.50	3205.40
bip52p	2200	7997	200	24305	24726	1.73	70531	3590.51	timeout
bip52u	2200	7997	200	231	236	2.16	3288104	2389.26	timeout
bip62p	1200	10002	200	22561	23121	2.48	35876	235.78	timeout
bip62u	1200	10002	200	216	222	2.78	4233548	1389.14	timeout
bipa2p	3300	18073	300	34763	35874	3.20	13379	111.43	timeout
bipa2u	3300	18073	300	331	341	3.02	855500	3415.45	timeout
bipe2p	550	5013	50	5616	5616	0.00	5247	46.57	52.36
bipe2u	550	5013	50	54	54	0.00	93	0.79	1.08
cc10-2p	1024	5120	135	34413	35790	4.00	35821	2183.72	timeout
cc10-2u	1024	5120	135	334	345	3.29	23308	32.94	timeout
cc11-2p	2048	11263	244	62001	64748	4.43	6581	3568.51	timeout
cc11-2u	2048	11263	244	602	621	3.16	3467	89.57	timeout
cc12-2p	4096	24574	473	118383	123894	4.66	370	682.57	timeout
cc12-2u	4096	24574	473	1149	1187	3.31	146	866.81	timeout
cc3-10p	1000	13500	50	11979	13070	9.11	22314	16.14	timeout
cc3-10u	1000	13500	50	117	126	7.69	43935	145.70	timeout
cc3-11p	1331	19965	61	14622	15928	8.93	6129	2146.95	timeout
cc3-11u	1331	19965	61	143	153	6.99	4960	1868.03	timeout
cc3-12p	1728	28512	74	17644	19312	9.45	5383	2924.59	timeout
cc3-12u	1728	28512	74	173	186	7.51	11502	551.07	timeout
cc3-4p	64	288	8	2338	2338	0.00	59325	0.17	49.86
cc3-4u	64	288	8	23	23	0.00	6720	0.01	8.70
cc3-5p	125	750	13	3454	3661	5.99	173453	23.80	timeout
cc3-5u	125	750	13	33	36	9.09	198738	0.06	timeout
cc5-3p	243	1215	27	7196	7303	1.49	455495	1250.72	timeout
cc5-3u	243	1215	27	71	71	0.00	626587	2.64	memout
cc6-2p	64	192	12	3271	3271	0.00	534	0.12	10.72
cc6-2u	64	192	12	32	32	0.00	33	0.01	13.84
cc6-3p	729	4368	76	20183	20570	1.92	82320	1417.54	timeout
cc6-3u	729	4368	76	197	197	0.00	18788	779.14	788.53
cc7-3p	2187	15308	222	55156	58111	5.36	2722	1857.41	timeout
cc7-3u	2187	15308	222	536	558	4.10	1601	124.21	timeout
cc9-2p	512	2304	64	16881	17486	3.58	128435	3462.74	timeout
cc9-2u	512	2304	64	164	169	3.05	166102	1023.15	timeout
hc6p	64	192	32	4003	4003	0.00	4300	0.03	2.10
hc6u	64	192	32	39	39	0.00	1167	0.06	0.38
hc7p	128	448	64	7830	7908	1.00	159145	0.63	timeout
hc7u	128	448	64	77	77	0.00	933186	0.06	1663.47
hc8p	256	1024	128	15204	15329	0.82	379356	3449.24	timeout
hc8u	256	1024	128	145	148	2.07	314424	0.61	memout
hc9p	512	2304	256	29961	30280	1.06	99023	2653.89	timeout
hc9u	512	2304	256	286	292	2.10	191157	3.97	timeout
hc10p	1024	5120	512	59266	60871	2.71	12667	408.40	timeout
hc10u	1024	5120	512	564	583	3.37	895223	2179.97	timeout
hc11p	2048	11264	1024	117408	121467	3.46	3215	285.22	timeout
hc11u	2048	11264	1024	1119	1170	4.56	95809	341.42	timeout
hc12p	4096	24576	2048	232907	240379	3.21	47	1477.29	timeout
hc12u	4096	24576	2048	2218	2318	4.51	15098	1195.46	timeout

Table 10. I640 STP instances. Results computed by B&C using the (x, y)-model (time limit = 1 hr., CPLEX memory limit = 12GB)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
i640-001	640	960	9	4033	4033	0.00	0	0.13	0.26
i640-002	640	960	9	3588	3588	0.00	0	0.10	0.28
i640-003	640	960	9	3438	3438	0.00	0	0.03	0.15
i640-004	640	960	9	4000	4000	0.00	0	0.03	0.20
i640-005	640	960	9	4006	4006	0.00	0	0.05	0.16
i640-011	640	4135	9	2392	2392	0.00	0	0.11	1.09
i640-012	640	4135	9	2465	2465	0.00	23	3.66	4.86
i640-013	640	4135	9	2399	2399	0.00	9	1.64	1.65
i640-014	640	4135	9	2171	2171	0.00	0	0.11	0.54
i640-015	640	4135	9	2347	2347	0.00	36	0.54	2.34
i640-021	640	204480	9	1749	1749	0.00	0	112.05	112.46
i640-022	640	204480	9	1756	1756	0.00	0	130.81	131.21
i640-023	640	204480	9	1754	1754	0.00	0	144.20	144.72
i640-024	640	204480	9	1751	1751	0.00	0	123.61	124.08
i640-025	640	204480	9	1745	1745	0.00	0	122.88	123.33
i640-031	640	1280	9	3278	3278	0.00	0	0.07	0.29
i640-032	640	1280	9	3187	3187	0.00	0	0.26	0.32
i640-033	640	1280	9	3260	3260	0.00	0	0.61	0.61
i640-034	640	1280	9	2953	2953	0.00	0	0.02	0.33
i640-035	640	1280	9	3292	3292	0.00	0	0.05	0.40
i640-041	640	40896	9	1897	1897	0.00	0	67.80	67.88
i640-042	640	40896	9	1934	1934	0.00	274	50.50	52.76
i640-043	640	40896	9	1931	1931	0.00	208	47.41	52.60
i640-044	640	40896	9	1938	1938	0.00	216	30.81	39.83
i640-045	640	40896	9	1866	1866	0.00	0	13.12	13.18
i640-101	640	960	25	8764	8764	0.00	0	0.70	0.71
i640-102	640	960	25	9109	9109	0.00	0	0.31	0.34
i640-103	640	960	25	8819	8819	0.00	0	0.37	0.37
i640-104	640	960	25	9040	9040	0.00	0	0.25	0.25
i640-105	640	960	25	9623	9623	0.00	0	0.59	1.28
i640-111	640	4135	25	6167	6167	0.00	501	1.62	15.75
i640-112	640	4135	25	6304	6304	0.00	631	13.59	21.15
i640-113	640	4135	25	6249	6249	0.00	1667	62.99	83.81
i640-114	640	4135	25	6308	6308	0.00	422	10.73	29.98
i640-115	640	4135	25	6217	6217	0.00	2456	36.99	37.81
i640-121	640	204480	25	4906	4906	0.00	0	215.90	249.40
i640-122	640	204480	25	4911	4911	0.00	0	11.87	232.78
i640-123	640	204480	25	4913	4913	0.00	0	646.41	646.83
i640-124	640	204480	25	4906	4906	0.00	0	371.54	371.94
i640-125	640	204480	25	4920	4920	0.00	0	659.71	660.17
i640-131	640	1280	25	8097	8097	0.00	0	1.11	1.12
i640-132	640	1280	25	8154	8154	0.00	0	0.27	2.30
i640-133	640	1280	25	8021	8021	0.00	0	0.27	0.83
i640-134	640	1280	25	7754	7754	0.00	0	1.08	1.09
i640-135	640	1280	25	7696	7696	0.00	0	0.10	0.79
i640-141	640	40896	25	5199	5199	0.00	3687	492.71	570.14
i640-142	640	40896	25	5193	5193	0.00	1517	439.67	528.03
i640-143	640	40896	25	5194	5194	0.00	1340	28.17	268.73
i640-144	640	40896	25	5205	5205	0.00	1563	45.42	315.65
i640-145	640	40896	25	5218	5218	0.00	3575	578.65	694.29

Table 11. I640 STP instances. Results computed by B&C using the (x, y)-model (time limit = 1 hr., CPLEX memory limit = 12GB)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
i640-201	640	960	50	16079	16079	0.00	0	0.34	0.34
i640-202	640	960	50	16324	16324	0.00	0	0.26	0.26
i640-203	640	960	50	16124	16124	0.00	0	0.30	0.31
i640-204	640	960	50	16239	16239	0.00	0	0.40	0.40
i640-205	640	960	50	16616	16616	0.00	0	0.64	0.65
i640-211	640	4135	50	11916	12048	1.11	113700	3087.86	timeout
i640-212	640	4135	50	11795	11795	0.00	5724	144.47	159.60
i640-213	640	4135	50	11879	11879	0.00	8837	175.93	289.39
i640-214	640	4135	50	11898	11898	0.00	93825	2442.67	2652.87
i640-215	640	4135	50	12081	12081	0.00	59008	1509.75	1570.79
i640-221	640	204480	50	9821	9821	0.00	316	828.17	2359.82
i640-222	640	204480	50	9798	9798	0.00	194	1247.22	1290.90
i640-223	640	204480	50	9811	9811	0.00	207	1227.49	1534.93
i640-224	640	204480	50	9805	9805	0.00	259	12.32	692.58
i640-225	640	204480	50	9807	9807	0.00	169	1621.67	1840.56
i640-231	640	1280	50	15014	15014	0.00	11	3.33	5.41
i640-232	640	1280	50	14630	14630	0.00	0	1.03	2.54
i640-233	640	1280	50	14797	14797	0.00	0	1.27	6.17
i640-234	640	1280	50	15203	15203	0.00	0	0.45	0.45
i640-235	640	1280	50	14803	14803	0.00	73	36.02	37.67
i640-241	640	40896	50	10186	10250	0.64	22719	100.65	timeout
i640-242	640	40896	50	10195	10195	0.00	9193	1797.15	2851.38
i640-243	640	40896	50	10215	10215	0.00	4097	153.26	636.13
i640-244	640	40896	50	10183	10274	0.90	25702	2493.22	timeout
i640-245	640	40896	50	10223	10223	0.00	16286	2621.95	3465.51
i640-301	640	960	160	45005	45005	0.00	0	0.48	0.48
i640-302	640	960	160	45736	45736	0.00	0	0.90	0.91
i640-303	640	960	160	44922	44922	0.00	0	0.24	0.34
i640-304	640	960	160	46233	46233	0.00	0	0.79	0.79
i640-305	640	960	160	45902	45902	0.00	0	1.01	1.02
i640-311	640	4135	160	35412	35839	1.21	95777	1654.27	timeout
i640-312	640	4135	160	35406	36021	1.74	104027	3517.68	timeout
i640-313	640	4135	160	35341	35543	0.57	89648	3413.68	timeout
i640-314	640	4135	160	35249	35659	1.17	115616	2828.63	timeout
i640-315	640	4135	160	35436	35898	1.30	119021	834.22	timeout
i640-321	640	204480	160	30988	31106	0.38	531	1685.39	timeout
i640-322	640	204480	160	30992	31068	0.25	700	3558.85	timeout
i640-323	640	204480	160	31009	31099	0.29	407	3346.03	timeout
i640-324	640	204480	160	31001	31096	0.31	786	1544.47	timeout
i640-325	640	204480	160	31006	31081	0.24	921	2635.04	timeout
i640-331	640	1280	160	42796	42796	0.00	17	14.22	14.47
i640-332	640	1280	160	42548	42548	0.00	32	19.69	20.04
i640-333	640	1280	160	42345	42345	0.00	59	34.13	34.18
i640-334	640	1280	160	42768	42768	0.00	436	7.90	9.69
i640-335	640	1280	160	43035	43035	0.00	210	15.82	15.99
i640-341	640	40896	160	31869	32116	0.78	6011	137.89	timeout
i640-342	640	40896	160	31839	32029	0.60	6892	124.02	timeout
i640-343	640	40896	160	31851	32096	0.77	9191	1409.95	timeout
i640-344	640	40896	160	31847	32052	0.65	7371	168.67	timeout
i640-345	640	40896	160	31841	32055	0.67	6741	197.94	timeout

Table 12. B and C STP instances (random graphs). Results computed by applying exact B&C on the (x, y)-model, (one hour timelimit)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
b01	50	63	9	82	82	0.00	0	0.04	0.08
b02	50	63	13	83	83	0.00	0	0.04	0.07
b03	50	63	25	138	138	0.00	0	0.04	0.08
b04	50	100	9	59	59	0.00	0	0.06	0.10
b05	50	100	13	61	61	0.00	0	0.03	0.08
b06	50	100	25	122	122	0.00	0	0.10	0.10
b07	75	94	13	111	111	0.00	0	0.06	0.10
b08	75	94	19	104	104	0.00	0	0.02	0.07
b09	75	94	38	220	220	0.00	0	0.04	0.08
b10	75	150	13	86	86	0.00	0	0.04	0.08
b11	75	150	19	88	88	0.00	0	0.03	0.11
b12	75	150	38	174	174	0.00	0	0.04	0.08
b13	100	125	17	165	165	0.00	0	0.08	0.08
b14	100	125	25	235	235	0.00	0	0.06	0.12
b15	100	125	50	318	318	0.00	0	0.03	0.08
b16	100	200	17	127	127	0.00	0	0.05	0.10
b17	100	200	25	131	131	0.00	0	0.03	0.12
b18	100	200	50	218	218	0.00	0	0.06	0.15
c01	500	625	5	85	85	0.00	0	0.04	0.21
c02	500	625	10	144	144	0.00	0	0.04	0.20
c03	500	625	83	754	754	0.00	0	0.40	0.40
c04	500	625	125	1079	1079	0.00	0	0.49	0.52
c05	500	625	250	1579	1579	0.00	0	0.37	0.39
c06	500	1000	5	55	55	0.00	0	0.07	0.31
c07	500	1000	10	102	102	0.00	0	0.04	0.26
c08	500	1000	83	509	509	0.00	0	0.59	0.59
c09	500	1000	125	707	707	0.00	0	0.63	0.66
c10	500	1000	250	1093	1093	0.00	0	0.79	0.79
c11	500	2500	5	32	32	0.00	0	0.14	0.83
c12	500	2500	10	46	46	0.00	0	0.21	0.58
c13	500	2500	83	258	258	0.00	23	0.29	4.71
c14	500	2500	125	323	323	0.00	0	0.89	2.32
c15	500	2500	250	556	556	0.00	0	0.51	4.59
c16	500	12500	5	11	11	0.00	0	0.24	1.02
c17	500	12500	10	18	18	0.00	0	0.18	1.71
c18	500	12500	83	113	113	0.00	0	0.34	3.23
c19	500	12500	125	146	146	0.00	0	0.38	2.71
c20	500	12500	250	267	267	0.00	0	0.61	2.89

Table 13. STP Random (D, E) – exact B&C (1 hr., (x, y)-model))

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
d01	1000	1250	5	106	106	0.00	0	0.16	0.47
d02	1000	1250	10	220	220	0.00	0	0.09	0.40
d03	1000	1250	167	1565	1565	0.00	0	0.28	0.73
d04	1000	1250	250	1935	1935	0.00	0	0.83	0.86
d05	1000	1250	500	3250	3250	0.00	0	0.66	0.67
d06	1000	2000	5	67	67	0.00	0	0.85	0.89
d07	1000	2000	10	103	103	0.00	0	0.07	0.40
d08	1000	2000	167	1072	1072	0.00	0	1.55	2.30
d09	1000	2000	250	1448	1448	0.00	0	1.61	1.70
d10	1000	2000	500	2110	2110	0.00	0	1.72	1.73
d11	1000	5000	5	29	29	0.00	0	1.34	1.43
d12	1000	5000	10	42	42	0.00	0	0.03	1.10
d13	1000	5000	167	500	500	0.00	0	1.58	7.27
d14	1000	5000	250	667	667	0.00	0	2.87	15.63
d15	1000	5000	500	1116	1116	0.00	0	2.93	21.39
d16	1000	25000	5	13	13	0.00	0	0.11	2.31
d17	1000	25000	10	23	23	0.00	0	0.39	3.53
d18	1000	25000	167	223	223	0.00	0	9.41	9.56
d19	1000	25000	250	310	310	0.00	0	1.59	13.65
d20	1000	25000	500	537	537	0.00	0	2.69	17.38
e01	2500	3125	5	111	111	0.00	0	0.25	1.32
e02	2500	3125	10	214	214	0.00	0	0.41	0.87
e03	2500	3125	417	4013	4013	0.00	0	2.81	2.83
e04	2500	3125	625	5101	5101	0.00	0	0.76	0.78
e05	2500	3125	1250	8128	8128	0.00	0	1.28	1.33
e06	2500	5000	5	73	73	0.00	0	1.41	1.43
e07	2500	5000	10	145	145	0.00	0	1.72	2.71
e08	2500	5000	417	2640	2640	0.00	0	4.41	4.43
e09	2500	5000	625	3604	3604	0.00	0	4.82	4.84
e10	2500	5000	1250	5600	5600	0.00	0	3.31	3.32
e11	2500	12500	5	34	34	0.00	0	1.28	3.71
e12	2500	12500	10	67	67	0.00	0	1.74	4.80
e13	2500	12500	417	1280	1280	0.00	0	75.57	75.60
e14	2500	12500	625	1732	1732	0.00	0	113.05	123.28
e15	2500	12500	1250	2784	2784	0.00	0	28.73	62.08
e16	2500	62500	5	15	15	0.00	0	0.51	15.04
e17	2500	62500	10	25	25	0.00	0	3.83	7.38
e18	2500	62500	417	564	564	0.00	148	216.43	216.65
e19	2500	62500	625	758	758	0.00	0	51.82	62.59
e20	2500	62500	1250	1342	1342	0.00	0	40.51	53.83

Table 14. STP ALUE and ALUT – exact B&C (1 hr., (x, y)-model))

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
alue2087	1244	1971	34	1049	1049	0.00	0	3.04	4.35
alue2105	1220	1858	34	1032	1032	0.00	0	0.86	2.57
alue3146	3626	5869	64	2240	2240	0.00	0	21.48	59.62
alue5067	3524	5560	68	2586	2586	0.00	0	53.02	53.04
alue5345	5179	8165	68	3507	3507	0.00	0	700.46	700.51
alue5623	4472	6938	68	3413	3413	0.00	5	255.36	371.84
alue5901	11543	18429	68	3912	3912	0.00	0	2394.47	2449.25
alue6179	3372	5213	67	2452	2452	0.00	0	5.15	50.82
alue6457	3932	6137	68	3057	3057	0.00	0	32.98	159.32
alue6735	4119	6696	68	2696	2696	0.00	0	16.49	76.08
alue6951	2818	4419	67	2386	2386	0.00	0	7.28	37.09
alue7065	34046	54841	544	23277	24085	3.47	0	67.25	timeout
alue7066	6405	10454	16	2256	2256	0.00	0	2426.34	3431.84
alue7080	34479	55494	2344	61334	62699	2.23	0	150.89	timeout
alue7229	940	1474	34	824	824	0.00	0	0.06	2.19
alut0787	1160	2089	34	982	982	0.00	0	4.11	7.01
alut0805	966	1666	34	958	958	0.00	0	1.82	6.73
alut1181	3041	5693	64	2353	2353	0.00	0	169.95	357.22
alut2010	6104	11011	68	3307	3307	0.00	0	441.00	562.01
alut2288	9070	16595	68	3843	3843	0.00	0	1250.81	1552.97
alut2566	5021	9055	68	3073	3073	0.00	0	38.85	635.93
alut2610	33901	62816	204	11791	12369	4.90	0	86.85	timeout
alut2625	36711	68117	879	34171	35681	4.42	0	220.69	timeout
alut2764	387	626	34	640	640	0.00	0	0.19	0.72

Table 15. STP DIW – exact B&C (1 hr., (x, y)-model))

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
diw0234	5349	10086	25	1996	1996	0.00	0	12.03	172.20
diw0250	353	608	11	350	350	0.00	0	0.04	0.41
diw0260	539	985	12	468	468	0.00	0	0.04	0.81
diw0313	468	822	14	397	397	0.00	0	0.05	0.66
diw0393	212	381	11	302	302	0.00	0	0.04	0.23
diw0445	1804	3311	33	1363	1363	0.00	0	0.15	30.86
diw0459	3636	6789	25	1362	1362	0.00	0	3.47	32.55
diw0460	339	579	13	345	345	0.00	0	0.06	0.45
diw0473	2213	4135	25	1098	1098	0.00	0	8.20	13.14
diw0487	2414	4386	25	1424	1424	0.00	0	20.55	22.46
diw0495	938	1655	10	616	616	0.00	0	0.08	2.73
diw0513	918	1684	10	604	604	0.00	0	0.43	4.22
diw0523	1080	2015	10	561	561	0.00	0	0.06	1.83
diw0540	286	465	10	374	374	0.00	0	0.03	0.29
diw0559	3738	7013	18	1570	1570	0.00	0	38.80	429.24
diw0778	7231	13727	24	2173	2173	0.00	0	27.68	2265.90
diw0779	11821	22516	50	3925	4461	13.66	0	155.05	timeout
diw0795	3221	5938	10	1550	1550	0.00	0	29.14	536.39
diw0801	3023	5575	10	1587	1587	0.00	0	0.30	786.69
diw0819	10553	20066	32	3363	3399	1.07	0	25.69	timeout
diw0820	11749	22384	37	3619	4182	15.56	0	759.16	timeout

Table 16. STP DMXA – exact B&C (1 hr., (x, y)-model))

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
dmxa0296	233	386	12	344	344	0.00	0	0.04	0.33
dmxa0368	2050	3676	18	1017	1017	0.00	0	22.40	24.33
dmxa0454	1848	3286	16	914	914	0.00	0	6.77	28.11
dmxa0628	169	280	10	275	275	0.00	0	0.04	0.45
dmxa0734	663	1154	11	506	506	0.00	0	0.05	2.07
dmxa0848	499	861	16	594	594	0.00	0	2.15	2.16
dmxa0903	632	1087	10	580	580	0.00	0	0.06	5.86
dmxa1010	3983	7108	23	1488	1488	0.00	0	9.27	203.04
dmxa1109	343	559	17	454	454	0.00	0	0.76	0.76
dmxa1200	770	1383	21	750	750	0.00	0	0.05	4.56
dmxa1304	298	503	10	311	311	0.00	0	0.17	0.33
dmxa1516	720	1269	11	508	508	0.00	0	0.23	1.57
dmxa1721	1005	1731	18	780	780	0.00	0	2.80	2.80
dmxa1801	2333	4137	17	1365	1365	0.00	0	85.33	120.83

Table 17. STP GAP – exact B&C (1 hr., (x, y)-model))

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
gap1307	342	552	17	549	549	0.00	0	0.02	0.45
gap1413	541	906	10	457	457	0.00	0	0.05	0.69
gap1500	220	374	17	254	254	0.00	0	0.01	0.13
gap1810	429	702	17	482	482	0.00	0	0.22	0.65
gap1904	735	1256	21	763	763	0.00	0	0.06	2.73
gap2007	2039	3548	17	1104	1104	0.00	3	9.73	26.20
gap2119	1724	2975	29	1244	1244	0.00	0	24.46	24.47
gap2740	1196	2084	14	745	745	0.00	0	0.07	12.74
gap2800	386	653	12	386	386	0.00	0	0.03	0.80
gap2975	179	293	10	245	245	0.00	0	0.04	0.20
gap3036	346	583	13	457	457	0.00	0	0.04	1.81
gap3100	921	1558	11	640	640	0.00	0	0.72	6.84
gap3128	10393	18043	104	4240	4292	1.23	0	1308.85	timeout

Table 18. STP TAQ – exact B&C (1 hr., (x, y)-model))

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
taq0014	6466	11046	128	5326	5326	0.00	0	1394.95	1394.97
taq0023	572	963	11	621	621	0.00	0	2.34	2.35
taq0365	4186	7074	22	1914	1914	0.00	0	36.05	317.22
taq0377	6836	11715	136	6393	6393	0.00	0	1982.38	2580.12
taq0431	1128	1905	13	897	897	0.00	0	0.09	15.23
taq0631	609	932	10	581	581	0.00	0	0.03	2.45
taq0739	837	1438	16	848	848	0.00	0	6.94	10.89
taq0741	712	1217	16	847	847	0.00	0	5.27	7.33
taq0751	1051	1791	16	939	939	0.00	0	0.09	11.29
taq0891	331	560	10	319	319	0.00	0	0.19	0.49
taq0903	6163	10490	130	5099	5099	0.00	0	2946.15	2946.18
taq0910	310	514	17	370	370	0.00	0	0.01	0.25
taq0920	122	194	17	210	210	0.00	0	0.06	0.16
taq0978	777	1239	10	566	566	0.00	0	0.09	1.50

Table 19. STP LIN – exact B&C (1 hr., (x, y)-model))

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
lin01	53	80	4	503	503	0.00	0	0.02	0.05
lin02	55	82	6	557	557	0.00	0	0.01	0.05
lin03	57	84	8	926	926	0.00	0	0.01	0.05
lin04	157	266	6	1239	1239	0.00	0	0.03	0.16
lin05	160	269	9	1703	1703	0.00	0	0.01	0.11
lin06	165	274	14	1348	1348	0.00	0	0.01	0.17
lin07	307	526	6	1885	1885	0.00	0	0.03	0.37
lin08	311	530	10	2248	2248	0.00	0	0.01	0.14
lin09	313	532	12	2752	2752	0.00	0	0.23	0.24
lin10	321	540	20	4132	4132	0.00	0	0.17	0.81
lin11	816	1460	10	4280	4280	0.00	0	5.84	5.85
lin12	818	1462	12	5250	5250	0.00	0	2.64	10.08
lin13	822	1466	16	4609	4609	0.00	0	0.38	2.61
lin14	828	1472	22	5824	5824	0.00	0	0.02	3.10
lin15	840	1484	34	7145	7145	0.00	0	0.38	1.73
lin16	1981	3633	12	6618	6618	0.00	0	4.74	84.71
lin17	1989	3641	20	8405	8405	0.00	0	3.12	87.73
lin18	1994	3646	25	9714	9714	0.00	0	98.85	98.86
lin19	2010	3662	41	13268	13268	0.00	0	37.32	56.69
lin20	3675	6709	11	6673	6673	0.00	0	60.24	179.30
lin21	3683	6717	20	9143	9143	0.00	0	131.81	169.90
lin22	3692	6726	28	10519	10519	0.00	0	13.99	199.19
lin23	3716	6750	52	17560	17560	0.00	0	1233.00	1347.57
lin24	7998	14734	16	15013	15076	0.42	0	1271.55	timeout
lin25	8007	14743	24	17803	17803	0.00	0	2479.91	3137.25
lin26	8013	14749	30	21594	21801	0.96	0	56.07	timeout
lin27	8017	14753	36	20340	20719	1.86	0	2013.30	timeout
lin28	8062	14798	81	31649	32667	3.22	0	60.54	timeout
lin29	19083	35636	24	23042	23956	3.97	0	877.08	timeout
lin30	19091	35644	31	26799	27750	3.55	0	3557.65	timeout
lin31	19100	35653	40	30342	32062	5.67	0	74.99	timeout
lin32	19112	35665	53	38076	40109	5.34	0	82.77	timeout
lin33	19177	35730	117	54350	56631	4.20	0	126.54	timeout
lin34	38282	71521	34	42693	45463	6.49	0	98.16	timeout
lin35	38294	71533	45	48135	51227	6.42	0	2.38	timeout
lin36	38307	71546	58	52279	56103	7.31	0	1671.04	timeout
lin37	38418	71657	172	95716	100254	4.74	0	212.67	timeout

Table 20. STP MSM – exact B&C (1 hr., (x, y)-model))

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
msm0580	338	541	11	467	467	0.00	0	0.43	0.80
msm0654	1290	2270	10	823	823	0.00	0	0.10	11.99
msm0709	1442	2403	16	884	884	0.00	0	0.07	8.27
msm0920	752	1264	26	806	806	0.00	0	0.53	4.14
msm1008	402	695	11	494	494	0.00	0	0.05	1.57
msm1234	933	1632	13	550	550	0.00	0	0.05	4.21
msm1477	1199	2078	31	1068	1068	0.00	0	0.76	3.54
msm1707	278	478	11	564	564	0.00	0	0.06	0.35
msm1844	90	135	10	188	188	0.00	0	0.20	0.20
msm1931	875	1522	10	604	604	0.00	0	0.05	5.36
msm2000	898	1562	10	594	594	0.00	0	0.59	6.72
msm2152	2132	3702	37	1590	1590	0.00	0	7.37	43.39
msm2326	418	723	14	399	399	0.00	0	0.18	0.52
msm2492	4045	7094	12	1459	1459	0.00	0	0.27	419.45
msm2525	3031	5239	12	1290	1290	0.00	0	0.20	50.11
msm2601	2961	5100	16	1440	1440	0.00	0	63.88	103.42
msm2705	1359	2458	13	714	714	0.00	0	0.11	11.77
msm2802	1709	2963	18	926	926	0.00	0	2.44	11.73
msm2846	3263	5783	89	3135	3135	0.00	0	300.71	300.73
msm3277	1704	2991	12	869	869	0.00	0	0.13	16.82
msm3676	957	1554	10	607	607	0.00	0	0.05	2.34
msm3727	4640	8255	21	1376	1376	0.00	0	0.37	78.60
msm3829	4221	7255	12	1571	1571	0.00	0	199.90	199.93
msm4038	237	390	11	353	353	0.00	0	0.08	0.39
msm4114	402	690	16	393	393	0.00	0	0.03	0.40
msm4190	391	666	16	381	381	0.00	0	0.13	0.50
msm4224	191	302	11	311	311	0.00	0	0.04	0.22
msm4312	5181	8893	10	2016	2016	0.00	0	916.04	1023.96
msm4414	317	476	11	408	408	0.00	0	0.01	0.30
msm4515	777	1358	13	630	630	0.00	0	0.06	3.76

Table 21. STP Vienna instances (after adv. preprocessing). Results computed by B&C on the (x, y)-model (one hour timelimit). Starting solution generated through the partitioning heuristic.

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
G101a	10734	16345	96	3467833	3493090	0.73	0	3095.17	timeout
G103a	36270	57370	2930	19758256	19953249	0.99	0	51.89	timeout
G105a	14586	22450	525	12466204	12514342	0.39	0	29.17	timeout
G106a	62618	100067	5373	43269003	44578604	3.03	0	101.82	timeout
G107a	15536	23858	893	7321683	7327721	0.08	0	3633.30	timeout
G201a	8286	12617	188	3484029	3484028	0.00	0	1030.57	1030.67
G202a	14028	21610	985	6849423	6849423	0.00	0	1739.26	1789.35
G203a	25651	40610	1999	13105484	13168788	0.48	0	3058.50	timeout
G204a	9939	15249	376	5313548	5313548	0.00	0	613.06	659.28
G205a	37398	59323	3146	24630914	24833434	0.82	0	73.96	timeout
G206a	13688	21197	789	9175622	9175622	0.00	0	2324.95	2348.78
G207a	7565	11521	98	2265834	2265834	0.00	0	1079.39	1081.26
G301a	13291	20261	181	4780637	4798737	0.38	0	19.25	timeout
G302a	24951	38647	1797	13239324	13313814	0.56	0	2337.02	timeout
G303a	37085	57711	2915	27786753	27965620	0.64	0	45.75	timeout
G304a	15213	23329	403	6711198	6725387	0.21	0	2727.51	timeout
G305a	47016	73861	3809	40367482	40666186	0.74	0	65.75	timeout
G306a	55423	87779	4766	32893144	33983262	3.31	0	94.38	timeout
G308a	13298	20351	86	4635438	4702025	1.44	0	2330.12	timeout
G309a	18704	28851	868	11233173	11266010	0.29	0	2173.60	timeout

Table 22. STP Vienna instances (after adv. preprocessing). Results computed by B&C on the (x, y)-model (one hour timelimit). Starting solution generated through the partitioning heuristic.

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
I001a	14675	22055	941	253921201	253921201	0.00	0	1045.85	1139.21
I002a	23800	35758	1282	399790333	399812639	0.01	0	3069.05	timeout
I003a	16270	23919	2336	788774494	788774494	0.00	3	2502.37	2515.23
I004a	867	1238	263	279512692	279512692	0.00	0	2.30	3.56
I005a	1677	2430	491	390876350	390876350	0.00	0	3.20	3.21
I006a	13339	19532	1842	504526035	504526035	0.00	13	1073.21	1076.40
I007a	6873	10299	599	177909660	177909660	0.00	0	9.58	68.33
I008a	6522	9629	708	201788202	201788202	0.00	0	10.49	167.01
I009a	14977	22435	1053	275558727	275558727	0.00	0	651.71	651.77
I010a	13041	19545	782	207889674	207889674	0.00	0	239.17	244.73
I011a	9298	13685	1202	317589880	317589880	0.00	0	166.60	166.69
I012a	3500	5214	387	118893243	118893243	0.00	0	3.44	14.61
I013a	7147	10608	670	193190339	193190339	0.00	0	141.66	141.69
I014a	3577	5311	364	105173465	105173465	0.00	0	4.30	13.13
I015a	20573	30541	2119	592228432	592244798	0.00	0	20.39	timeout
I016a	27214	39824	3434	1110860598	1110927444	0.01	0	39.27	timeout
I017a	7571	11571	386	109739695	109739695	0.00	0	104.34	104.37
I018a	12258	18014	1549	463887832	463887832	0.00	0	435.92	436.00
I019a	11693	17624	732	217647693	217647693	0.00	0	774.24	830.95
I020a	6405	9564	508	146515460	146515460	0.00	0	96.58	96.61
I021a	5195	7861	295	106470644	106470644	0.00	0	55.88	56.47
I022a	8869	13551	356	106799980	106799980	0.00	0	5.46	573.97
I023a	13724	20863	403	131044872	131044872	0.00	0	868.18	868.25
I024a	32357	48250	2511	757174824	758495435	0.17	0	29.72	timeout
I025a	10055	14961	833	232790758	232790758	0.00	0	2330.53	2522.76
I026a	18155	26568	2661	928032223	928032223	0.00	0	2840.63	2892.30
I027a	40772	60555	3490	975633120	976819170	0.12	0	45.52	timeout
I028a	43690	66461	1597	381864370	384056368	0.57	0	17.33	timeout
I029a	32979	49627	1946	490533383	492204633	0.34	0	27.43	timeout
I030a	12941	19279	1093	321646787	321646787	0.00	0	312.64	313.64
I031a	21054	31410	1832	578279525	578286011	0.00	0	3444.67	timeout
I032a	21345	31353	2454	773096651	773096651	0.00	0	1311.03	1311.29
I033a	8500	12700	548	134461857	134461857	0.00	0	78.93	78.97
I034a	9128	13668	606	165115148	165115148	0.00	0	330.78	360.40
I035a	13129	19420	1428	414440370	414440370	0.00	0	30.98	301.05
I036a	17036	25482	1258	375260864	375260864	0.00	62	3104.09	3130.81
I037a	5886	8869	392	105720727	105720727	0.00	0	110.14	110.16
I038a	7733	11478	798	255767543	255767543	0.00	7	269.53	276.57
I039a	3719	5533	306	85566290	85566290	0.00	0	5.75	29.26
I040a	18837	28156	1501	431446659	431503742	0.01	0	23.03	timeout
I041a	22466	33868	1014	301838450	301917055	0.03	0	17.19	timeout
I042a	23925	35806	1923	532017577	532135202	0.02	0	31.14	timeout
I043a	4511	6740	335	95722094	95722094	0.00	0	36.29	83.13
I044a	31500	46757	2954	804402651	804536894	0.02	0	48.79	timeout
I045a	6775	10227	378	105944062	105944062	0.00	0	10.05	135.57
I046a	32376	48054	3154	925328946	925477525	0.02	0	43.62	timeout
I047a	10622	15440	1791	695163406	695163406	0.00	0	1319.66	1321.82
I048a	4920	7356	320	91509264	91509264	0.00	0	62.94	69.00
I049a	15045	22713	821	294811505	294811505	0.00	0	2512.69	2572.08

Table 23. Vienna STP instances (after adv. preprocessing). Results computed by B&C on the (x, y)-model (one hour timelimit). Starting solution generated through the partitioning heuristic.

name	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	LB	UB	Gap%	Nodes	Time	Time-t
I050a	17787	26176	2232	792554822	792613306	0.01	0	2411.56	timeout
I051a	12130	17892	1337	357230840	357230839	0.00	29	1088.87	1107.48
I052a	160	237	23	13309487	13309487	0.00	0	0.07	0.16
I053a	693	1023	102	30854904	30854904	0.00	0	0.24	0.98
I054a	540	817	25	15841596	15841596	0.00	0	0.23	0.58
I055a	4701	6979	483	144164924	144164924	0.00	0	7.78	46.51
I056a	290	439	34	14171206	14171206	0.00	0	0.36	0.42
I057a	13078	19368	1346	412746415	412746415	0.00	7	655.33	655.45
I058a	7877	11657	997	305024188	305024188	0.00	0	130.72	131.14
I059a	2800	4157	286	107617854	107617854	0.00	0	3.21	10.86
I060a	18991	28536	1158	337243232	337300512	0.02	0	1581.16	timeout
I061a	20958	31465	1337	362974714	363046922	0.02	0	75.65	timeout
I062a	23714	35305	2812	792915560	792949801	0.00	0	3427.64	timeout
I063a	9600	14042	1291	459801704	459801704	0.00	0	360.85	360.90
I064a	31712	46711	3182	862984846	863120759	0.02	0	56.62	timeout
I065a	1185	1756	119	32965718	32965718	0.00	0	5.29	5.42
I066a	4551	6821	417	174219813	174219813	0.00	0	28.99	33.88
I067a	10318	15588	579	175540750	175540750	0.00	0	14.50	541.95
I068a	12191	18023	1302	420730046	420730046	0.00	0	286.25	286.31
I069a	3508	5156	452	135161583	135161583	0.00	0	9.55	45.68
I070a	6739	10064	511	136700139	136700139	0.00	0	106.93	113.25
I071a	12772	18886	1281	382539099	382539099	0.00	0	460.45	460.51
I072a	11628	17411	851	289019226	289019226	0.00	0	962.50	962.58
I073a	7510	10873	1337	663004987	663004987	0.00	0	230.53	231.59
I074a	4441	6562	548	165573383	165573383	0.00	0	8.49	33.79
I075a	23195	34362	2498	815334535	815405990	0.01	0	49.81	timeout
I076a	4909	7268	498	166249692	166249692	0.00	0	98.54	105.40
I077a	9153	13363	1490	472503150	472503150	0.00	0	421.47	421.51
I078a	5864	8662	692	185525490	185525490	0.00	0	55.55	67.79
I079a	7933	11807	497	150506933	150506933	0.00	0	768.11	777.41
I080a	7589	11256	499	164299652	164299652	0.00	0	237.93	238.00
I081a	10747	16029	751	247527679	247527679	0.00	0	1156.69	1156.78
I082a	5850	8693	435	147407632	147407632	0.00	0	96.69	103.98
I083a	34221	50301	4138	1405426943	1405601179	0.01	0	50.70	timeout
I084a	17050	25201	1918	627187559	627187559	0.00	0	2225.23	2225.52
I085a	2780	4123	243	80628079	80628079	0.00	0	11.93	11.95

Table 24. ES1000FST and ES10000FST STP instances. Results computed by B&C on the (x, y)-model (one hour timelimit). Starting solution generated through the partitioning heuristic.

name	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	LB	UB	Gap%	Nodes	Time	Time-t
es10000fst01	27019	39407	10000	712255860	716838421	0.64	0	86.08	timeout
es1000fst01	2865	4267	1000	230535806	230535806	0.00	9	152.90	153.08
es1000fst02	2629	3793	1000	227886471	227886471	0.00	3	65.07	65.09
es1000fst03	2762	4047	1000	227807756	227807756	0.00	0	55.74	55.76
es1000fst04	2778	4083	1000	230200846	230200846	0.00	0	76.82	79.69
es1000fst05	2676	3894	1000	228330602	228330602	0.00	2	73.44	73.47
es1000fst06	2815	4162	1000	231028456	231028456	0.00	15	155.35	155.64
es1000fst07	2604	3756	1000	230945624	230945623	0.00	0	50.66	50.67
es1000fst08	2834	4207	1000	230639115	230639115	0.00	3	148.22	150.50
es1000fst09	2846	4187	1000	227745838	227745838	0.00	3	112.22	114.63
es1000fst10	2546	3620	1000	229267101	229267101	0.00	0	36.53	36.82
es1000fst11	2763	4038	1000	231605619	231605619	0.00	3	65.24	70.25
es1000fst12	2984	4484	1000	230904712	230904712	0.00	18	151.04	151.89
es1000fst13	2532	3615	1000	228031092	228031092	0.00	0	46.73	57.13
es1000fst14	2840	4200	1000	234318491	234318491	0.00	27	127.89	130.17
es1000fst15	2733	3997	1000	229965775	229965775	0.00	0	61.07	70.51

Table 25. TSPFST STP instances. Results computed by B&C on the (x, y)-model (one hour timelimit). Starting solution generated through the partitioning heuristic.

name	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	LB	UB	Gap%	Nodes	Time	Time-t
a280fst	313	328	279	2502	2502	0.00	0	0.06	0.13
att48fst	139	202	48	30236	30236	0.00	0	0.26	0.26
att532fst	1468	2152	532	84009	84009	0.00	3	16.65	16.79
berlin52fst	89	104	52	6760	6760	0.00	0	0.04	0.10
bier127fst	258	357	127	104284	104284	0.00	0	0.31	0.32
d1291fst	1365	1456	1291	481421	481421	0.00	0	0.78	0.78
d1655fst	1906	2083	1655	584948	584948	0.00	0	0.59	3.26
d198fst	232	256	198	129175	129175	0.00	0	0.08	0.11
d2103fst	2206	2272	2103	769797	769797	0.00	0	0.53	1.59
d493fst	1055	1473	493	320137	320137	0.00	0	6.62	6.62
d657fst	1416	1978	657	471589	471589	0.00	6	12.54	13.57
dsj1000fst	2562	3655	1000	17564659	17564659	0.00	0	12.79	12.81
eil101fst	330	538	101	605	605	0.00	0	2.59	2.59
eil51fst	181	289	51	409	409	0.00	0	0.47	0.49
eil76fst	237	378	76	513	513	0.00	0	1.02	1.03
fl1400fst	2694	4546	1400	17965021	17980676	0.09	300487	739.74	timeout
fl1577fst	2413	3412	1577	19825626	19825626	0.00	0	44.49	50.67
fl3795fst	4859	6539	3795	25514345	25529877	0.06	242873	3635.34	timeout
fl417fst	732	1084	417	10883190	10883190	0.00	61	2.10	2.80
fnl4461fst	17127	27352	4461	180014	182747	1.52	0	87.73	timeout
gil262fst	537	723	262	2306	2306	0.00	0	0.70	0.95
kroA100fst	197	250	100	20401	20401	0.00	0	0.09	0.17
kroA150fst	389	562	150	25700	25700	0.00	0	0.69	0.75
kroA200fst	500	714	200	28652	28652	0.00	0	0.22	1.03
kroB100fst	230	313	100	21211	21211	0.00	0	0.20	0.27
kroB150fst	420	619	150	25217	25217	0.00	0	0.29	1.23
kroB200fst	480	670	200	28803	28803	0.00	0	0.75	1.40
kroC100fst	244	337	100	20492	20492	0.00	0	0.33	0.33
kroD100fst	216	288	100	20437	20437	0.00	0	0.18	0.18
kroE100fst	226	306	100	21245	21245	0.00	0	0.18	0.18

Table 26. STP TSPFST – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
lin105fst	216	323	105	13430	13429	0.01	0	0.29	0.29
lin318fst	678	1030	318	39335	39335	0.00	0	0.53	2.90
linhp318fst	678	1030	318	39335	39335	0.00	0	0.55	2.72
nrw1379fst	5096	8105	1379	56207	56207	0.00	33	1098.61	1101.99
p654fst	777	867	654	314925	314925	0.00	0	0.19	0.53
pcb1173fst	1912	2223	1173	53301	53301	0.00	0	1.39	3.81
pcb3038fst	5829	7552	3038	131895	131895	0.00	0	122.51	122.54
pcb442fst	503	531	442	47675	47675	0.00	0	0.10	0.25
pla7397fst	8790	9815	7397	22481625	22481625	0.00	0	57.91	77.42
pr1002fst	1473	1715	1002	243176	243176	0.00	0	0.40	1.67
pr107fst	111	110	107	34850	34850	0.00	0	0.01	0.03
pr124fst	154	165	124	52759	52759	0.00	0	0.03	0.07
pr136fst	196	250	136	86811	86811	0.00	0	0.02	0.07
pr144fst	221	285	144	52925	52925	0.00	0	0.09	0.21
pr152fst	308	431	152	64323	64323	0.00	0	0.21	0.23
pr226fst	255	269	226	70700	70700	0.00	0	0.01	0.11
pr2392fst	3398	3966	2392	358989	358989	0.00	0	0.98	5.21
pr264fst	280	287	264	41400	41400	0.00	0	0.03	0.07
pr299fst	420	500	299	44671	44671	0.00	0	0.13	0.30
pr439fst	572	662	439	97400	97400	0.00	0	0.22	0.55
pr76fst	168	247	76	95908	95908	0.00	0	0.24	0.25
rat195fst	560	870	195	2386	2386	0.00	0	2.24	2.25
rat575fst	1986	3176	575	6808	6808	0.00	3	94.11	94.13
rat783fst	2397	3715	783	8883	8883	0.00	0	136.38	136.40
rat99fst	269	399	99	1225	1225	0.00	0	0.34	0.45
rd100fst	201	253	100	764269099	764269099	0.00	0	0.15	0.15
rd400fst	1001	1419	400	1490972006	1490972006	0.00	0	4.40	4.68
rl11849fst	13963	15315	11849	8779590	8779590	0.00	0	98.88	102.95
rl1304fst	1562	1694	1304	236649	236649	0.00	0	1.08	1.14
rl1323fst	1598	1750	1323	253620	253620	0.00	0	1.88	1.88
rl1889fst	2382	2674	1889	295208	295208	0.00	0	7.62	8.41
rl5915fst	6569	6980	5915	533226	533226	0.00	0	1.57	14.76
rl5934fst	6827	7365	5934	529890	529890	0.00	0	3.59	22.53
st70fst	133	169	70	626	626	0.00	0	0.03	0.09
ts225fst	225	224	225	1120	1120	80.00	0	0.01	0.03
tsp225fst	242	252	225	356850	356850	0.00	0	0.07	0.08
u1060fst	1835	2429	1060	21265372	21265372	0.00	771	11.53	12.86
u1432fst	1432	1431	1432	1465	1465	0.00	0	0.05	0.37
u159fst	184	186	159	390	390	0.00	0	0.03	0.11
u1817fst	1831	1846	1817	5513053	5513053	0.00	0	0.05	0.68
u2152fst	2167	2184	2152	6253305	6253305	0.00	0	0.91	1.07
u2319fst	2319	2318	2319	2322	2322	0.00	0	0.05	0.74
u574fst	990	1258	574	3509275	3509275	0.00	0	1.88	2.06
u724fst	1180	1537	724	4069628	4069628	0.00	0	0.49	3.73
vm1084fst	1679	2058	1084	2248390	2248390	0.00	0	5.64	6.19

Exact Results for the PCSTP

Table 27. PCSTP Random – exact B&C (1 hr., (x, y)-model))

name	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	LB	UB	Gap%	Nodes	Time	Time-t
a0200RandGraph.1.2	200	1636	200	122.21	122.21	0.00	0	0.15	0.16
a0200RandGraph.1.5	200	1575	200	141.88	141.88	0.00	0	0.19	0.19
a0200RandGraph.2	200	1605	200	157.02	157.02	0.00	0	0.21	0.21
a0200RandGraph.3	200	1616	200	170.29	170.29	0.00	0	0.17	0.17
a0400RandGraph.1.2	400	3194	400	234.98	234.98	0.00	0	0.39	0.40
a0400RandGraph.1.5	400	3231	400	272.87	272.87	0.00	0	0.45	0.46
a0400RandGraph.2	400	3292	400	300.92	300.92	0.00	0	0.48	0.49
a0400RandGraph.3	400	3222	400	337.60	337.60	0.00	0	0.32	0.33
a0600RandGraph.1.2	600	4821	600	360.39	360.39	0.00	0	0.63	0.64
a0600RandGraph.1.5	600	4845	600	407.63	407.63	0.00	0	0.72	0.73
a0600RandGraph.2	600	4831	600	460.02	460.02	0.00	0	0.72	0.90
a0600RandGraph.3	600	4808	600	507.94	507.94	0.00	0	0.51	0.52
a0800RandGraph.1.2	800	6453	800	464.78	464.78	0.00	0	3.12	3.18
a0800RandGraph.1.5	800	6301	800	530.44	530.44	0.00	0	0.96	0.98
a0800RandGraph.2	800	6465	800	603.33	603.33	0.00	0	0.86	0.88
a0800RandGraph.3	800	6385	800	663.61	663.61	0.00	0	1.13	1.15
a1000RandGraph.1.2	1000	8067	1000	580.35	580.35	0.00	0	1.46	1.48
a1000RandGraph.1.5	1000	7868	1000	673.47	673.47	0.00	0	1.12	1.43
a1000RandGraph.2	1000	8201	1000	753.29	753.29	0.00	0	1.22	1.25
a1000RandGraph.3	1000	8107	1000	831.58	831.58	0.00	0	1.03	1.06
a1200RandGraph.1.2	1200	9448	1200	705.67	705.67	0.00	0	3.73	3.81
a1200RandGraph.1.5	1200	9625	1200	810.48	810.48	0.00	0	1.51	1.54
a1200RandGraph.2	1200	9546	1200	906.79	906.79	0.00	0	1.51	1.54
a1200RandGraph.3	1200	9451	1200	1012.45	1012.45	0.00	0	1.08	1.10
a1400RandGraph.1.2	1400	11192	1400	810.63	810.63	0.00	0	4.30	4.32
a1400RandGraph.1.5	1400	11226	1400	938.93	938.93	0.00	0	1.69	1.72
a1400RandGraph.2	1400	11100	1400	1051.01	1051.01	0.00	0	1.83	1.86
a1400RandGraph.3	1400	11263	1400	1158.96	1158.96	0.00	0	1.25	1.28
a1600RandGraph.1.2	1600	12869	1600	943.74	943.74	0.00	0	7.90	7.97
a1600RandGraph.1.5	1600	12739	1600	1078.80	1078.80	0.00	0	2.87	2.90
a1600RandGraph.2	1600	12779	1600	1217.05	1217.05	0.00	0	2.08	2.12
a1600RandGraph.3	1600	12963	1600	1351.98	1351.98	0.00	0	1.65	1.68
a1800RandGraph.1.2	1800	14473	1800	1061.39	1061.39	0.00	0	10.14	10.27
a1800RandGraph.1.5	1800	14222	1800	1218.78	1218.78	0.00	0	2.27	2.31
a1800RandGraph.2	1800	14329	1800	1364.89	1364.89	0.00	0	2.35	2.39
a1800RandGraph.3	1800	14531	1800	1507.27	1507.27	0.00	0	1.76	1.80

Table 28. PCSTP Random – exact B&C (1 hr., (x, y)-model))

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
a2000RandGraph.1.2	2000	16008	2000	1151.95	1151.95	0.00	0	3.30	3.34
a2000RandGraph.1.5	2000	15835	2000	1330.77	1330.77	0.00	0	2.47	3.63
a2000RandGraph.2	2000	16062	2000	1483.84	1483.84	0.00	0	2.70	2.75
a2000RandGraph.3	2000	15751	2000	1669.35	1669.35	0.00	0	1.96	2.00
a3000RandGraph.1.2	3000	24045	3000	1781.19	1781.19	0.00	0	15.14	16.60
a3000RandGraph.1.5	3000	23852	3000	2028.62	2028.62	0.00	0	3.98	4.05
a3000RandGraph.2	3000	24065	3000	2282.92	2282.92	0.00	0	3.67	3.73
a3000RandGraph.3	3000	24026	3000	2537.20	2537.20	0.00	0	4.16	4.23
a4000RandGraph.1.2	4000	32087	4000	2396.92	2396.92	0.00	0	43.66	45.06
a4000RandGraph.1.5	4000	32119	4000	2735.18	2735.18	0.00	0	17.70	17.78
a4000RandGraph.2	4000	31880	4000	3072.26	3072.26	0.00	0	5.52	5.62
a4000RandGraph.3	4000	32025	4000	3406.62	3406.62	0.00	0	5.88	5.96
a6000RandGraph.1.2	6000	47899	6000	3544.39	3544.39	0.00	0	64.92	65.03
a6000RandGraph.1.5	6000	48077	6000	4059.19	4059.19	0.00	0	57.71	57.81
a6000RandGraph.2	6000	48069	6000	4551.77	4551.77	0.00	0	10.52	10.67
a6000RandGraph.3	6000	47915	6000	5049.26	5049.26	0.00	0	7.64	7.76
a8000RandGraph.1.2	8000	64373	8000	4719.97	4719.97	0.00	0	192.83	203.87
a8000RandGraph.1.5	8000	63812	8000	5394.57	5394.57	0.00	0	159.41	159.59
a8000RandGraph.2	8000	63874	8000	6055.13	6055.13	0.00	0	14.07	14.26
a8000RandGraph.3	8000	64177	8000	6710.62	6710.62	0.00	0	15.37	15.60
a10000RandGraph.1.2	10000	80298	10000	5927.32	5927.32	0.00	0	114.48	114.74
a10000RandGraph.1.5	10000	80288	10000	6775.55	6775.55	0.00	0	99.70	99.92
a10000RandGraph.2	10000	79908	10000	7594.38	7594.38	0.00	0	17.75	17.98
a10000RandGraph.3	10000	79778	10000	8422.56	8422.56	0.00	0	18.82	19.05
a12000RandGraph.1.2	12000	96093	12000	7073.95	7073.95	0.00	0	224.08	224.26
a12000RandGraph.1.5	12000	96391	12000	8084.13	8084.13	0.00	0	365.52	365.73
a12000RandGraph.2	12000	95987	12000	9064.24	9064.24	0.00	0	54.51	54.76
a12000RandGraph.3	12000	96449	12000	10061.82	10061.82	0.00	0	52.63	52.94
a14000RandGraph.1.2	14000	112016	14000	8271.47	8271.47	0.00	0	460.85	464.97
a14000RandGraph.1.5	14000	112228	14000	9475.59	9475.59	0.00	0	232.10	232.43
a14000RandGraph.2	14000	112369	14000	10639.20	10639.20	0.00	0	31.40	31.78
a14000RandGraph.3	14000	111869	14000	11776.89	11776.89	0.00	0	60.53	60.93

Table 29. PCSTP Hand ICERM small – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
handsi01	39600	78704	39600	295.45	295.45	0.00	0	53.31	53.54
handsi02	39600	78704	2077	125.43	125.43	0.00	0	1941.39	1941.62
handsi03	39600	78704	39600	56.15	56.15	0.00	0	45.72	45.90
handsi04	39600	78704	4056	720.29	724.89	0.64	0	1248.70	timeout
handsi05	39600	78704	39600	35.04	35.04	0.00	0	1.19	40.66
handsi06	39600	78704	3989	452.95	452.95	0.00	0	729.38	729.66
handsi07	39600	78704	39600	18.41	18.41	0.00	0	1.35	62.39
handsi08	39600	78704	3970	229.53	229.53	0.00	0	373.00	961.06
handsi09	39600	78704	39600	5.90	5.98	1.33	0	1.38	timeout
handsi10	39600	78704	3811	1803.70	1803.70	0.00	0	528.21	530.05

Table 30. PCSTP Hand ICERM big – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
handbi01	158400	315808	158400	1345.18	1360.20	1.12	0	6.96	timeout
handbi02	158400	315808	8594	516.55	533.03	3.19	0	8.79	timeout
handbi03	158400	315808	158400	243.13	243.13	0.00	0	880.23	881.74
handbi04	158400	315808	16297	3053.83	3231.93	5.83	0	9.11	timeout
handbi05	158400	315808	158400	184.47	184.47	0.00	0	704.64	705.79
handbi06	158400	315808	16022	2849.14	2946.26	3.41	0	8.78	timeout
handbi07	158400	315808	158400	150.97	150.97	0.00	0	1013.30	1014.05
handbi08	158400	315808	15544	2237.62	2279.64	1.88	0	2898.53	timeout
handbi09	158400	315808	158400	107.77	107.77	0.00	0	746.91	749.91
handbi10	158400	315808	15879	1867.64	1876.61	0.48	0	3197.11	timeout
handbi11	158400	315808	158400	68.88	68.95	0.11	0	8.10	timeout
handbi12	158400	315808	15642	138.26	138.26	0.00	0	56.81	2209.74
handbi13	158400	315808	158400	4.16	4.28	3.02	0	1175.68	timeout
handbi14	158400	315808	15971	7881.77	7881.77	0.00	0	7.77	42.65

Table 31. PCSTP Hand DIMACS small – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
handsd01	42500	84475	42500	171.64	171.64	0.00	0	73.70	73.99
handsd02	42500	84475	2325	156.77	160.72	2.52	0	1.23	timeout
handsd03	42500	84475	42500	31.31	31.31	0.00	0	42.96	43.22
handsd04	42500	84475	4368	485.98	494.72	1.80	0	1.28	timeout
handsd05	42500	84475	42500	21.94	21.94	0.00	0	55.73	56.01
handsd06	42500	84475	4158	279.90	279.90	0.00	0	1231.42	1231.61
handsd07	42500	84475	42500	11.80	11.80	0.00	0	55.70	55.89
handsd08	42500	84475	4392	143.24	143.24	0.00	0	359.63	638.85
handsd09	42500	84475	42500	3.80	3.82	0.57	0	1.31	timeout
handsd10	42500	84475	4179	1034.77	1034.77	0.00	0	1.15	16.66

Table 32. PCSTP Hand DIMACS big – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
handbd01	169800	338551	169800	721.02	730.24	1.28	0	7.79	timeout
handbd02	169800	338551	8886	281.41	297.04	5.56	0	7.13	timeout
handbd03	169800	338551	169800	135.07	135.07	0.00	0	607.12	608.32
handbd04	169800	338551	16981	1718.08	1829.87	6.51	0	7.68	timeout
handbd05	169800	338551	169800	105.47	105.47	0.00	0	761.78	797.94
handbd06	169800	338551	16447	1459.48	1537.22	5.33	0	8.87	timeout
handbd07	169800	338551	169800	77.86	77.86	0.00	0	2635.29	2636.58
handbd08	169800	338551	17486	1343.42	1375.71	2.40	0	8.80	timeout
handbd09	169800	338551	169800	62.72	62.72	0.00	0	1017.34	1018.57
handbd10	169800	338551	17293	1124.96	1144.06	1.70	0	8.30	timeout
handbd11	169800	338551	169800	46.77	46.77	0.00	0	720.39	722.77
handbd12	169800	338551	16466	320.48	321.23	0.23	0	2876.29	timeout
handbd13	169800	338551	169800	12.87	13.22	2.74	0	8.37	timeout
handbd14	169800	338551	17597	4379.10	4379.10	0.00	0	8.62	38.95

Table 33. PCSTP CRR (C) – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
C01-A	500	625	5	18	18	0.00	0	0.010	0.109
C01-B	500	625	5	85	85	0.00	0	0.030	0.117
C02-A	500	625	10	50	50	0.00	0	0.009	0.120
C02-B	500	625	10	141	141	0.00	0	0.029	0.116
C03-A	500	625	83	414	414	0.00	0	0.144	0.156
C03-B	500	625	83	737	737	0.00	0	0.210	0.522
C04-A	500	625	125	618	618	0.00	0	0.204	0.214
C04-B	500	625	125	1063	1063	0.00	0	0.350	0.362
C05-A	500	625	250	1080	1080	0.00	0	0.235	0.307
C05-B	500	625	250	1528	1528	0.00	0	0.342	0.385
C06-A	500	1000	5	18	18	0.00	0	0.011	0.140
C06-B	500	1000	5	55	55	0.00	0	0.035	0.189
C07-A	500	1000	10	50	50	0.00	0	0.011	0.159
C07-B	500	1000	10	102	102	0.00	0	0.043	0.149
C08-A	500	1000	83	361	361	0.00	0	0.273	0.287
C08-B	500	1000	83	500	500	0.00	0	0.306	0.356
C09-A	500	1000	125	533	533	0.00	0	0.349	0.371
C09-B	500	1000	125	694	694	0.00	0	0.340	0.643
C10-A	500	1000	250	859	859	0.00	0	0.356	0.368
C10-B	500	1000	250	1069	1069	0.00	0	0.481	0.655
C11-A	500	2500	5	18	18	0.00	0	0.019	0.251
C11-B	500	2500	5	32	32	0.00	0	0.102	0.448
C12-A	500	2500	10	38	38	0.00	0	0.170	0.491
C12-B	500	2500	10	46	46	0.00	0	0.142	0.387
C13-A	500	2500	83	236	236	0.00	0	0.657	1.208
C13-B	500	2500	83	258	258	0.00	0	0.703	1.423
C14-A	500	2500	125	293	293	0.00	0	0.739	0.774
C14-B	500	2500	125	318	318	0.00	0	0.604	0.987
C15-A	500	2500	250	501	501	0.00	0	1.078	1.323
C15-B	500	2500	250	551	551	0.00	0	0.519	1.220
C16-A	500	12500	5	11	11	0.00	0	0.346	1.215
C16-B	500	12500	5	11	11	0.00	0	0.499	1.313
C17-A	500	12500	10	18	18	0.00	0	0.941	2.234
C17-B	500	12500	10	18	18	0.00	0	0.983	2.204
C18-A	500	12500	83	111	111	0.00	0	3.325	3.957
C18-B	500	12500	83	113	113	0.00	0	3.892	4.203
C19-A	500	12500	125	146	146	0.00	0	3.692	3.754
C19-B	500	12500	125	146	146	0.00	0	3.365	3.422
C20-A	500	12500	250	266	266	0.00	0	3.498	3.566
C20-B	500	12500	250	267	267	0.00	0	2.158	2.965

Table 34. PCSTP CRR (D) – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
D01-A	1000	1250	5	18	18	0.00	0	0.014	0.159
D01-B	1000	1250	5	106	106	0.00	0	0.042	0.291
D02-A	1000	1250	10	50	50	0.00	0	0.013	0.263
D02-B	1000	1250	10	218	218	0.00	0	0.098	0.268
D03-A	1000	1250	167	807	807	0.00	0	0.334	0.351
D03-B	1000	1250	167	1509	1509	0.00	0	0.334	0.455
D04-A	1000	1250	250	1203	1203	0.00	0	0.370	0.384
D04-B	1000	1250	250	1881	1881	0.00	0	0.450	1.058
D05-A	1000	1250	500	2157	2157	0.00	0	0.480	0.495
D05-B	1000	1250	500	3135	3135	0.00	0	0.744	1.180
D06-A	1000	2000	5	18	18	0.00	0	0.017	0.299
D06-B	1000	2000	5	67	67	0.00	0	0.347	0.449
D07-A	1000	2000	10	50	50	0.00	0	0.017	0.286
D07-B	1000	2000	10	103	103	0.00	0	0.085	0.298
D08-A	1000	2000	167	755	755	0.00	0	0.802	0.822
D08-B	1000	2000	167	1036	1036	0.00	0	0.604	1.489
D09-A	1000	2000	250	1070	1070	0.00	0	0.893	0.912
D09-B	1000	2000	250	1420	1420	0.00	0	0.838	1.701
D10-A	1000	2000	500	1671	1671	0.00	0	2.487	2.507
D10-B	1000	2000	500	2079	2079	0.00	0	3.794	3.818
D11-A	1000	5000	5	18	18	0.00	0	0.036	0.507
D11-B	1000	5000	5	29	29	0.00	0	0.182	0.866
D12-A	1000	5000	10	42	42	0.00	0	0.246	0.772
D12-B	1000	5000	10	42	42	0.00	0	0.254	0.707
D13-A	1000	5000	167	445	445	0.00	0	1.398	3.802
D13-B	1000	5000	167	486	486	0.00	0	2.012	3.768
D14-A	1000	5000	250	602	602	0.00	0	1.729	11.767
D14-B	1000	5000	250	665	665	0.00	0	1.856	3.330
D15-A	1000	5000	500	1042	1042	0.00	0	5.156	5.189
D15-B	1000	5000	500	1108	1108	0.00	0	5.353	5.385
D16-A	1000	25000	5	13	13	0.00	0	1.273	3.722
D16-B	1000	25000	5	13	13	0.00	0	1.429	3.482
D17-A	1000	25000	10	23	23	0.00	0	1.684	5.042
D17-B	1000	25000	10	23	23	0.00	0	2.028	4.703
D18-A	1000	25000	167	218	218	0.00	0	11.754	11.867
D18-B	1000	25000	167	223	223	0.00	0	10.587	10.701
D19-A	1000	25000	250	306	306	0.00	0	13.832	13.973
D19-B	1000	25000	250	310	310	0.00	0	9.669	9.809
D20-A	1000	25000	500	536	536	0.00	0	8.909	9.023
D20-B	1000	25000	500	537	537	0.00	0	7.586	7.731

Table 35. PCSTP JMP – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
K100	100	351	14	135759	135759	0.18	0	0.008	0.125
K100.1	100	348	11	125674	125674	1.25	0	0.007	0.142
K100.10	100	319	14	143327	143327	6.81	0	0.008	0.140
K100.2	100	339	13	200262	200262	3.36	0	0.039	0.260
K100.3	100	407	10	118160	115953	1.87	0	0.007	0.145
K100.4	100	364	10	87498	87498	0.00	0	0.018	0.097
K100.5	100	358	16	119078	119078	0.00	0	0.082	0.091
K100.6	100	307	11	132886	132886	0.00	0	0.022	0.107
K100.7	100	315	13	172457	172457	0.00	0	0.118	0.127
K100.8	100	343	15	210869	210869	0.00	0	0.106	0.144
K100.9	100	333	11	124475	124475	1.25	0	0.007	0.114
K200	200	691	33	329211	329211	0.00	0	0.250	0.261
K400	400	1515	62	350093	350093	0.00	0	0.746	0.774
K400.1	400	1470	64	490771	490771	0.00	0	1.356	1.372
K400.10	400	1507	49	394191	394191	0.00	0	2.102	2.117
K400.2	400	1527	61	477073	477073	0.00	0	2.703	2.718
K400.3	400	1492	55	415328	415328	0.00	0	0.953	0.969
K400.4	400	1426	55	389451	389451	0.00	0	1.232	1.248
K400.5	400	1456	76	519526	519526	0.00	0	2.205	2.220
K400.6	400	1576	55	374849	374849	0.00	0	0.924	0.940
K400.7	400	1442	67	474466	474466	0.00	0	1.538	1.777
K400.8	400	1516	60	418614	418614	0.00	0	1.504	1.524
K400.9	400	1500	53	383105	383105	0.00	0	1.587	1.603
P100	100	317	33	803300	803300	0.00	0	0.044	0.112
P100.1	100	284	32	926238	926238	0.00	0	0.082	0.095
P100.2	100	297	26	401641	401641	0.00	0	0.026	0.110
P100.3	100	316	24	659644	659644	0.00	0	0.032	0.087
P100.4	100	284	32	827419	827419	0.00	0	0.035	0.111
P200	200	587	48	1317874	1317874	0.00	0	0.180	0.198
P400	400	1200	94	2459904	2459904	0.00	0	0.267	0.354
P400.1	400	1212	120	2808440	2808440	0.00	0	0.851	0.866
P400.2	400	1196	107	2518577	2518577	0.00	0	0.400	0.486
P400.3	400	1175	113	2951725	2951725	0.00	0	0.565	0.590
P400.4	400	1144	94	2852956	2852956	0.00	0	0.366	0.378

Table 36. PCSTP H – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
hc6p	64	192	64	3908	3908	0.00	4337	3.05	3.12
hc6u	64	192	64	36	36	0.00	3	0.10	0.10
hc7p	128	448	128	7657	7724	0.88	723863	1789.09	timeout
hc7u	128	448	128	72	72	0.00	226	0.14	0.20
hc8p	256	1024	256	15023	15278	1.70	352212	2555.40	timeout
hc8u	256	1024	256	143	143	0.00	90445	9.16	76.45
hc9p	512	2304	512	29654	30352	2.35	58392	3585.37	timeout
hc9u	512	2304	512	283	283	0.00	505974	448.95	643.90
hc10p	1024	5120	1024	58908	60433	2.59	8758	1174.11	timeout
hc10u	1024	5120	1024	555	561	1.08	430109	268.27	1316.36
hc11p	2048	11264	2048	116859	121336	3.83	777	409.26	timeout
hc11u	2048	11264	2048	1105	1119	1.27	228608	1271.99	2291.06
hc12p	4096	24576	4096	231958	240499	3.68	31	1578.95	timeout
hc12u	4096	24576	4096	2193	2226	1.50	57785	1838.04	timeout

Table 37. PCSTP H2 – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
hc6p2	64	192	64	3923	3923	0.00	13581	1.32	9.62
hc6u2	64	192	64	20	20	0.00	0	0.04	0.04
hc7p2	128	448	128	7711	7711	0.00	568249	1431.52	2099.48
hc7u2	128	448	128	47	47	0.00	162	0.09	0.17
hc8p2	256	1024	256	15057	15296	1.59	273561	2180.51	timeout
hc8u2	256	1024	256	97	97	0.00	46106	73.70	74.40
hc9p2	512	2304	512	29782	30523	2.49	64911	1551.65	timeout
hc9u2	512	2304	512	188	191	1.60	240160	17.54	timeout
hc10p2	1024	5120	1024	58853	60884	3.45	22571	547.58	timeout
hc10u2	1024	5120	1024	376	383	1.86	188563	855.43	timeout
hc11p2	2048	11264	2048	116788	121134	3.72	1361	1627.12	timeout
hc11u2	2048	11264	2048	745	760	2.12	255332	692.77	2282.83
hc12p2	4096	24576	4096	231990	240183	3.53	33	1807.80	timeout
hc12u2	4096	24576	4096	1478	1515	2.50	160370	2256.05	2368.53

Table 38. PCSTP ACTMODPC – exact B&C (1 hr., mice (x, y)-model, rest y-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
HCMV	3863	29293	3863	7371.54	7371.54	0.00	10	1.82	1.85
drosophila001	5226	93394	5226	8273.98	8273.98	0.00	2298	19.04	20.00
drosophila005	5226	93394	5226	8121.31	8121.31	0.00	4822	23.03	23.03
drosophila0075	5226	93394	5226	8039.86	8039.86	0.00	1044	15.92	15.92
lymphoma	2034	7756	2034	3341.89	3341.89	0.00	0	0.19	0.19
metabol_expr_mice_1	3523	4345	3523	11346.93	11346.93	0.00	0	1.42	1.43
metabol_expr_mice_2	3514	4332	3514	16250.24	16250.24	0.00	0	0.41	0.81
metabol_expr_mice_3	2853	3335	2853	16919.62	16919.62	0.00	0	0.68	1.02

Table 39. PCSTP PUCNU – exact B&C (1 hr., y-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
bip42nu	1200	3982	200	226	226	0.00	126291	100.30	111.97
bip52nu	2200	7997	200	222	222	0.00	487986	492.95	493.14
bip62nu	1200	10002	200	214	214	0.00	218494	1.22	213.92
bipa2nu	3300	18073	300	323	325	0.86	197262	93.31	memout
bipe2nu	550	5013	50	53	53	0.00	0	0.35	0.44
cc10-2nu	1024	5120	135	164	172	4.88	244628	7.37	memout
cc11-2nu	2048	11263	244	297	312	5.05	241166	295.05	memout
cc12-2nu	4096	24574	473	552	583	5.62	155526	696.43	memout
cc3-10nu	1000	13500	50	59	61	3.60	678509	0.71	memout
cc3-11nu	1331	19965	61	75	80	7.53	403377	2.28	timeout
cc3-12nu	1728	28512	74	89	96	7.87	337979	7.91	memout
cc3-4nu	64	288	8	10	10	0.00	0	0.01	0.04
cc3-5nu	125	750	13	17	17	0.00	0	0.01	0.07
cc5-3nu	243	1215	27	36	36	0.00	0	0.09	0.09
cc6-2nu	64	192	12	15	15	0.00	0	0.04	0.08
cc6-3nu	729	4368	76	95	95	0.00	21829	32.41	33.02
cc7-3nu	2187	15308	222	263	278	5.74	235833	16.62	memout
cc9-2nu	512	2304	64	83	83	0.00	75306	23.44	23.60

Table 40. PCSTP i640 – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
i640-001	640	960	9	2932	2932	0.00	0	0.03	0.21
i640-002	640	960	9	2795	2795	0.00	0	0.21	0.28
i640-003	640	960	9	2630	2630	0.00	0	0.30	0.30
i640-004	640	960	9	3356	3356	0.00	0	0.17	0.18
i640-005	640	960	9	3288	3288	0.00	0	0.04	0.18
i640-011	640	4135	9	2166	2166	0.00	18	3.14	3.15
i640-012	640	4135	9	1999	1999	0.00	0	1.28	1.41
i640-013	640	4135	9	2002	2002	0.00	0	0.80	0.81
i640-014	640	4135	9	2171	2171	0.00	0	0.09	0.68
i640-015	640	4135	9	2295	2295	0.00	0	0.53	0.73
i640-021	640	204480	9	1585	1585	0.00	0	102.47	102.98
i640-022	640	204480	9	1704	1704	0.00	0	104.62	104.92
i640-023	640	204480	9	1614	1614	0.00	0	136.38	136.92
i640-024	640	204480	9	1563	1563	0.00	0	131.38	131.77
i640-025	640	204480	9	1550	1550	0.00	0	139.86	140.26
i640-031	640	1280	9	2400	2400	0.00	0	0.06	0.26
i640-032	640	1280	9	2053	2053	0.00	0	0.18	0.22
i640-033	640	1280	9	2789	2789	0.00	0	0.10	0.35
i640-034	640	1280	9	2757	2757	0.00	0	0.06	0.28
i640-035	640	1280	9	2510	2510	0.00	0	0.05	0.39
i640-041	640	40896	9	1639	1639	0.00	0	19.17	19.22
i640-042	640	40896	9	1621	1621	0.00	0	7.58	8.09
i640-043	640	40896	9	1401	1401	0.00	0	6.43	6.54
i640-044	640	40896	9	1665	1665	0.00	0	0.45	9.17
i640-045	640	40896	9	1569	1569	0.00	0	8.43	9.96
i640-101	640	960	25	8135	8135	0.00	0	0.47	1.28
i640-102	640	960	25	7791	7791	0.00	0	0.30	0.32
i640-103	640	960	25	7854	7854	0.00	0	0.35	0.43
i640-104	640	960	25	6965	6965	0.00	0	0.33	0.33
i640-105	640	960	25	8669	8669	0.00	0	0.50	0.51
i640-111	640	4135	25	5323	5323	0.00	210	12.65	12.80
i640-112	640	4135	25	5908	5908	0.00	252	9.53	11.32
i640-113	640	4135	25	5886	5886	0.00	449	16.78	23.76
i640-114	640	4135	25	5630	5630	0.00	14	4.65	4.69
i640-115	640	4135	25	6040	6040	0.00	1767	33.76	37.46
i640-121	640	204480	25	4379	4379	0.00	3	4.40	184.45
i640-122	640	204480	25	4707	4707	0.00	60	206.43	212.37
i640-123	640	204480	25	4509	4509	0.00	6	200.89	229.18
i640-124	640	204480	25	4586	4586	0.00	33	270.80	274.24
i640-125	640	204480	25	4556	4556	0.00	34	268.08	275.59
i640-131	640	1280	25	6490	6490	0.00	0	0.56	0.57
i640-132	640	1280	25	7800	7800	0.00	0	0.57	1.31
i640-133	640	1280	25	6808	6808	0.00	0	0.68	1.47
i640-134	640	1280	25	6415	6415	0.00	0	0.31	0.32
i640-135	640	1280	25	6618	6618	0.00	0	0.38	1.54
i640-141	640	40896	25	5171	5171	0.00	2091	244.67	301.89
i640-142	640	40896	25	4733	4733	0.00	261	75.33	94.96
i640-143	640	40896	25	4321	4321	0.00	187	81.97	89.85
i640-144	640	40896	25	4562	4562	0.00	3593	310.46	368.83
i640-145	640	40896	25	4907	4907	0.00	1201	101.48	135.75

Table 41. PCSTP i640 – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
i640-201	640	960	50	14372	14372	0.00	0	0.34	0.35
i640-202	640	960	50	15059	15059	0.00	0	0.19	0.19
i640-203	640	960	50	13848	13848	0.00	0	0.20	0.31
i640-204	640	960	50	13108	13108	0.00	0	0.39	0.39
i640-205	640	960	50	15308	15308	0.00	0	1.37	1.37
i640-211	640	4135	50	11109	11109	0.00	17223	580.91	671.55
i640-212	640	4135	50	10351	10351	0.00	1730	18.34	78.31
i640-213	640	4135	50	10388	10388	0.00	3130	131.54	150.41
i640-214	640	4135	50	10675	10675	0.00	224	20.00	20.78
i640-215	640	4135	50	10740	10740	0.00	7992	227.13	245.83
i640-221	640	204480	50	8400	8400	0.00	379	812.25	990.76
i640-222	640	204480	50	8993	8993	0.00	98	562.99	685.81
i640-223	640	204480	50	9210	9210	0.00	99	555.99	622.38
i640-224	640	204480	50	8870	8870	0.00	148	531.60	731.26
i640-225	640	204480	50	8386	8386	0.00	178	641.46	785.16
i640-231	640	1280	50	14279	14279	0.00	74	8.62	11.04
i640-232	640	1280	50	13526	13526	0.00	6	0.90	4.38
i640-233	640	1280	50	12948	12948	0.00	0	2.77	3.37
i640-234	640	1280	50	13645	13645	0.00	0	1.98	2.08
i640-235	640	1280	50	12650	12650	0.00	0	0.96	0.96
i640-241	640	40896	50	9716	9716	0.00	11155	1431.74	1623.03
i640-242	640	40896	50	9250	9250	0.00	929	86.21	214.24
i640-243	640	40896	50	9315	9315	0.00	12563	2678.07	2883.43
i640-244	640	40896	50	8950	8950	0.00	12603	1271.57	1331.80
i640-245	640	40896	50	9448	9448	0.00	12311	2151.40	2422.58
i640-301	640	960	160	42701	42701	0.00	0	1.13	1.14
i640-302	640	960	160	42606	42606	0.00	0	0.69	1.13
i640-303	640	960	160	41286	41286	0.00	0	0.71	0.72
i640-304	640	960	160	42050	42050	0.00	0	0.67	0.68
i640-305	640	960	160	42798	42798	0.00	36	2.98	2.99
i640-311	640	4135	160	33248	33588	1.03	68074	3579.67	timeout
i640-312	640	4135	160	32491	32721	0.71	67335	3347.66	timeout
i640-313	640	4135	160	32235	32401	0.52	75924	3335.24	timeout
i640-314	640	4135	160	32646	32967	0.99	73167	1707.18	timeout
i640-315	640	4135	160	32394	32688	0.91	90125	1432.40	timeout
i640-321	640	204480	160	28685	28792	0.37	765	3544.25	timeout
i640-322	640	204480	160	28371	28493	0.43	1036	1303.50	timeout
i640-323	640	204480	160	28094	28155	0.22	1565	1308.45	timeout
i640-324	640	204480	160	28674	28757	0.29	406	1932.04	timeout
i640-325	640	204480	160	28306	28404	0.35	1291	1375.16	timeout
i640-331	640	1280	160	39315	39315	0.00	16	6.33	6.42
i640-332	640	1280	160	39030	39030	0.00	0	5.32	5.32
i640-333	640	1280	160	39775	39775	0.00	348	31.24	33.27
i640-334	640	1280	160	39338	39338	0.00	3	11.44	11.45
i640-335	640	1280	160	39601	39601	0.00	714	15.69	17.51
i640-341	640	40896	160	29524	29697	0.59	3285	90.32	timeout
i640-342	640	40896	160	29657	29844	0.63	3057	47.04	timeout
i640-343	640	40896	160	29907	30056	0.50	4206	48.63	timeout
i640-344	640	40896	160	29754	29977	0.75	5569	119.79	timeout
i640-345	640	40896	160	29809	30031	0.74	3666	3456.47	timeout

Exact Results for the RPCSTP

Table 42. RPCSTP Cologne1 – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
i01M1	748	6332	10	109271.50	109271.50	0.00	0	0.83	0.84
i01M2	748	6332	10	315925.31	315925.31	0.00	0	22.33	22.34
i01M3	748	6332	10	355625.41	355625.41	0.00	0	25.29	25.31
i02M1	749	6343	11	104065.80	104065.80	0.00	0	0.76	0.77
i02M2	749	6343	11	352538.82	352538.82	0.00	0	0.19	24.38
i02M3	749	6343	11	454365.93	454365.93	0.00	0	45.74	45.76
i03M1	751	6343	13	139749.41	139749.41	0.00	0	1.09	1.10
i03M2	751	6343	13	407834.23	407834.23	0.00	0	18.17	18.20
i03M3	751	6343	13	456125.49	456125.49	0.00	0	44.51	44.83
i04M2	741	6293	3	89920.84	89920.84	0.00	0	0.14	5.99
i04M3	741	6293	3	97148.79	97148.79	0.00	0	6.45	6.46
i05M1	741	6296	3	26717.20	26717.20	0.00	0	0.83	0.85
i05M2	741	6296	3	100269.62	100269.62	0.00	0	0.87	10.39
i05M3	741	6296	3	110351.16	110351.16	0.00	0	42.89	42.91

Table 43. RPCSTP Cologne2 – exact B&C (1 hr., (x, y)-model)

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
i01M2	1803	16743	9	355467.68	355467.68	0.00	0	0.19	16.43
i01M3	1803	16743	9	628833.61	628833.61	0.00	0	174.65	174.71
i01M4	1803	16743	9	773398.30	773398.30	0.00	0	194.90	194.94
i02M2	1804	16740	10	288946.83	288946.83	0.00	0	0.24	13.39
i02M3	1804	16740	10	419184.16	419184.16	0.00	0	73.45	73.49
i02M4	1804	16740	10	430034.26	430034.26	0.00	0	118.20	118.24
i03M2	1809	16762	15	459894.78	459894.78	0.00	0	13.29	16.31
i03M3	1809	16762	15	643062.02	643062.02	0.00	0	325.05	325.10
i03M4	1809	16762	15	677733.07	677733.07	0.00	0	228.57	228.62
i04M2	1801	16719	4	161700.55	161700.55	0.00	0	0.26	10.12
i04M3	1801	16719	4	245287.20	245287.20	0.00	0	52.15	52.19
i04M4	1801	16719	4	245287.20	245287.20	0.00	0	51.13	51.18
i05M2	1810	16794	13	571031.42	571031.42	0.00	0	0.26	14.60
i05M3	1810	16794	13	672403.14	672403.14	0.00	0	58.53	58.58
i05M4	1810	16794	13	713973.62	713973.62	0.00	0	48.67	48.71

Exact Results for the MWCS

Table 44. JMPALMK MWCS instances. Results computed through applying B&C using the y-model after transforming the instances into their PCSTP representation.

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
a-0.62-d-0.25-e-0.25	500	2597	500	460.58	460.58	0.00	0	0.02	0.99
a-0.62-d-0.25-e-0.5	500	2597	500	992.97	992.97	0.00	0	0.05	0.39
a-0.62-d-0.25-e-0.75	500	2597	500	1447.54	1447.54	0.00	0	0.02	0.22
a-0.62-d-0.5-e-0.25	500	2597	500	280.83	280.83	0.00	0	0.02	0.28
a-0.62-d-0.5-e-0.5	500	2597	500	655.62	655.62	0.00	0	0.02	0.31
a-0.62-d-0.5-e-0.75	500	2597	500	965.55	965.55	0.00	0	0.02	0.23
a-0.62-d-0.75-e-0.25	500	2597	500	171.63	171.63	0.00	0	0.02	0.21
a-0.62-d-0.75-e-0.5	500	2597	500	362.19	362.19	0.00	0	0.02	0.22
a-0.62-d-0.75-e-0.75	500	2597	500	490.62	490.62	0.00	0	0.02	0.24
a-1-d-0.25-e-0.25	500	6519	500	471.39	471.39	0.00	0	0.05	0.46
a-1-d-0.25-e-0.5	500	6519	500	995.31	995.31	0.00	0	0.05	0.41
a-1-d-0.25-e-0.75	500	6519	500	1447.54	1447.54	0.00	0	0.08	0.45
a-1-d-0.5-e-0.25	500	6519	500	286.92	286.92	0.00	0	0.04	0.44
a-1-d-0.5-e-0.5	500	6519	500	661.71	661.71	0.00	0	0.05	0.48
a-1-d-0.5-e-0.75	500	6519	500	965.55	965.55	0.00	0	0.05	0.47
a-1-d-0.75-e-0.25	500	6519	500	171.63	171.63	0.00	0	0.05	0.44
a-1-d-0.75-e-0.5	500	6519	500	362.19	362.19	0.00	0	0.05	0.47
a-1-d-0.75-e-0.75	500	6519	500	490.62	490.62	0.00	0	0.04	0.48
a-0.647-d-0.25-e-0.25	750	4219	750	702.64	702.64	0.00	0	0.03	0.83
a-0.647-d-0.25-e-0.5	750	4219	750	1419.78	1419.78	0.00	0	0.03	0.61
a-0.647-d-0.25-e-0.75	750	4219	750	2116.58	2116.58	0.00	0	0.04	0.39
a-0.647-d-0.5-e-0.25	750	4219	750	403.18	403.18	0.00	0	0.03	0.32
a-0.647-d-0.5-e-0.5	750	4219	750	946.13	946.13	0.00	0	0.03	0.32
a-0.647-d-0.5-e-0.75	750	4219	750	1382.77	1382.77	0.00	0	0.04	0.39
a-0.647-d-0.75-e-0.25	750	4219	750	266.98	266.98	0.00	0	0.03	0.33
a-0.647-d-0.75-e-0.5	750	4219	750	580.41	580.41	0.00	0	0.03	0.33
a-0.647-d-0.75-e-0.75	750	4219	750	764.16	764.16	0.00	0	0.04	0.33
a-1-d-0.25-e-0.25	750	9822	750	708.14	708.14	0.00	0	0.07	0.73
a-1-d-0.25-e-0.5	750	9822	750	1426.45	1426.45	0.00	0	0.07	0.63
a-1-d-0.25-e-0.75	750	9822	750	2116.58	2116.58	0.00	0	0.08	0.64
a-1-d-0.5-e-0.25	750	9822	750	403.18	403.18	0.00	0	0.08	0.68
a-1-d-0.5-e-0.5	750	9822	750	946.13	946.13	0.00	0	0.08	0.74
a-1-d-0.5-e-0.75	750	9822	750	1382.77	1382.77	0.00	0	0.07	0.74
a-1-d-0.75-e-0.25	750	9822	750	266.98	266.98	0.00	0	0.08	0.70
a-1-d-0.75-e-0.5	750	9822	750	580.41	580.41	0.00	0	0.07	0.71
a-1-d-0.75-e-0.75	750	9822	750	764.16	764.16	0.00	0	0.08	0.72

Table 45. JMPALMK MWCS instances. Results computed by B&C using the y-model after transforming the instances into their PCSTP representation.

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
a-0.6-d-0.25-e-0.25	1000	4936	1000	931.54	931.54	0.00	0	0.04	2.73
a-0.6-d-0.25-e-0.5	1000	4936	1000	1872.28	1872.28	0.00	0	0.04	0.88
a-0.6-d-0.25-e-0.75	1000	4936	1000	2789.58	2789.58	0.00	0	0.04	0.51
a-0.6-d-0.5-e-0.25	1000	4936	1000	522.53	522.53	0.00	0	0.04	0.93
a-0.6-d-0.5-e-0.5	1000	4936	1000	1197.85	1197.85	0.00	0	0.04	0.42
a-0.6-d-0.5-e-0.75	1000	4936	1000	1762.71	1762.71	0.00	0	0.04	0.43
a-0.6-d-0.75-e-0.25	1000	4936	1000	332.79	332.79	0.00	0	0.04	0.42
a-0.6-d-0.75-e-0.5	1000	4936	1000	754.30	754.30	0.00	0	0.04	0.70
a-0.6-d-0.75-e-0.75	1000	4936	1000	998.22	998.22	0.00	0	0.07	0.49
a-1-d-0.25-e-0.25	1000	13279	1000	939.39	939.39	0.00	0	0.10	1.06
a-1-d-0.25-e-0.5	1000	13279	1000	1883.21	1883.21	0.00	0	0.11	1.12
a-1-d-0.25-e-0.75	1000	13279	1000	2789.58	2789.58	0.00	0	0.11	1.00
a-1-d-0.5-e-0.25	1000	13279	1000	533.43	533.43	0.00	0	0.12	0.96
a-1-d-0.5-e-0.5	1000	13279	1000	1205.42	1205.42	0.00	0	0.10	1.11
a-1-d-0.5-e-0.75	1000	13279	1000	1770.28	1770.28	0.00	0	0.10	0.92
a-1-d-0.75-e-0.25	1000	13279	1000	336.83	336.83	0.00	0	0.11	0.95
a-1-d-0.75-e-0.5	1000	13279	1000	760.28	760.28	0.00	0	0.10	0.97
a-1-d-0.75-e-0.75	1000	13279	1000	1004.20	1004.20	0.00	0	0.13	1.11
a-0.6-d-0.25-e-0.25	1500	7662	1500	1333.48	1333.48	0.00	0	0.06	8.62
a-0.6-d-0.25-e-0.5	1500	7662	1500	2799.68	2799.68	0.00	0	0.06	1.55
a-0.6-d-0.25-e-0.75	1500	7662	1500	4230.25	4230.25	0.00	0	0.07	0.82
a-0.6-d-0.5-e-0.25	1500	7662	1500	847.45	847.45	0.00	0	0.06	0.65
a-0.6-d-0.5-e-0.5	1500	7662	1500	1858.09	1858.09	0.00	0	0.06	0.67
a-0.6-d-0.5-e-0.75	1500	7662	1500	2697.46	2697.46	0.00	0	0.06	1.66
a-0.6-d-0.75-e-0.25	1500	7662	1500	502.18	502.18	0.00	0	0.06	0.65
a-0.6-d-0.75-e-0.5	1500	7662	1500	1089.77	1089.77	0.00	0	0.06	0.68
a-0.6-d-0.75-e-0.75	1500	7662	1500	1423.61	1423.61	0.00	0	0.06	0.66
a-1-d-0.25-e-0.25	1500	20527	1500	1377.01	1377.01	0.00	0	0.17	1.86
a-1-d-0.25-e-0.5	1500	20527	1500	2820.05	2820.05	0.00	0	0.17	1.88
a-1-d-0.25-e-0.75	1500	20527	1500	4230.25	4230.25	0.00	0	0.20	1.73
a-1-d-0.5-e-0.25	1500	20527	1500	860.62	860.62	0.00	0	0.18	1.53
a-1-d-0.5-e-0.5	1500	20527	1500	1865.66	1865.66	0.00	0	0.18	1.90
a-1-d-0.5-e-0.75	1500	20527	1500	2707.70	2707.70	0.00	0	0.17	1.61
a-1-d-0.75-e-0.25	1500	20527	1500	502.18	502.18	0.00	0	0.16	1.51
a-1-d-0.75-e-0.5	1500	20527	1500	1089.77	1089.77	0.00	0	0.18	1.67
a-1-d-0.75-e-0.75	1500	20527	1500	1423.61	1423.61	0.00	0	0.19	1.85

Table 46. JMPALMK MWCS instances. Results computed by B&C after transforming the instances into their PCSTP representation. The model choice was left to the filter algorithm, i.e., the y-model was used for relatively dense graphs (drosophila001/005/0075/HCMV/lymphoma) and the (x, y)-model for the rest.

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
HCMV	3863	29293	3863	7371.54	7371.54	0.00	10	1.83	1.86
drosophila001	5226	93394	5226	8273.98	8273.98	0.00	2298	18.91	19.84
drosophila005	5226	93394	5226	8121.31	8121.31	0.00	4822	24.09	24.09
drosophila0075	5226	93394	5226	8039.86	8039.86	0.00	1044	13.95	13.95
lymphoma	2034	7756	2034	3341.89	3341.89	0.00	0	0.16	0.16
metabol_expr_mice_1	3523	4345	3523	11346.93	11346.93	0.00	0	1.50	1.51
metabol_expr_mice_2	3514	4332	3514	16250.24	16250.24	0.00	0	0.47	0.86
metabol_expr_mice_3	2853	3335	2853	16919.62	16919.62	0.00	0	0.73	1.07

Exact Results for the DCST

Table 47. TREEFARM DCST instances. Results computed by B&C using the (x, y)-model augmented with degree constraints. Instances not solved after the given timelimit of one hour are marked with timeout in the Time-t column. If infeasibility was proven during the timelimit, columns LB and UB contain the keyword infeasible.

name	$\|V\|$	$\|E\|$	$\|T\|$	LB	UB	Gap%	Nodes	Time	Time-t
TF101057-t1	52	1326	35	infeasible	infeasible	-	0	-	0.25
TF101057-t3	52	1326	35	2756	2756	0.00	0	0.47	0.48
TF101125-t1	304	46056	155	infeasible	infeasible	-	0	-	12.32
TF101125-t3	304	46056	155	55185	55185	0.00	94	2724.56	2730.31
TF101202-t1	188	17578	72	79920	79920	0.00	110	253.38	254.27
TF101202-t3	188	17578	72	77978	77978	0.00	106	472.98	479.80
TF102003-t1	832	345696	407	241032	-	-	0	-	timeout
TF102003-t3	832	345696	407	178410	474672	166.06	0	6.055	timeout
TF105035-t1	237	27966	104	35618	35618	0.00	80	474.73	474.91
TF105035-t3	237	27966	104	32916	32916	0.00	322	1548.81	1619.68
TF105272-t1	476	113050	223	186714	-	-	0	-	timeout
TF105272-t3	476	113050	223	125610	400546	218.88	0	1.96	timeout
TF105419-t1	55	1485	24	18668	18668	0.00	0	0.89	0.90
TF105419-t3	55	1485	24	18223	18223	0.00	2	3.62	3.63
TF105897-t1	314	49141	133	112852	-	-	0	-	timeout
TF105897-t3	314	49141	133	97320	200508	106.03	0	0.50	timeout
TF106403-t1	119	7021	46	54124	54124	0.00	6	11.05	11.39
TF106403-t3	119	7021	46	53760	53760	0.00	5	35.37	35.94
TF106478-t1	130	8385	54	55132	55132	0.00	9	10.51	10.63
TF106478-t3	130	8385	54	54839	54839	0.00	62	36.11	36.17

7 Detailed Heuristic Results

After applying B&C with a timeout of one hour to all instances, the instances that remained unsolved were selected to apply the heuristic setting. Each run has been computed by running the heuristic setting with ten different seeds, each with one hour timeout. The used heuristic approach was chosen dynamically through the filter algorithm. Each table is structured as follows: The column Instance gives the instance name, followed by the instance's number of nodes, edges and terminals.The next pair of columns (BEST) show objective value and time of computation for the best found solution. The following pairs (AVG and STD) list the average and standard deviation of these two values over all ten runs. For tests on the STP (PUC, I640, LIN, except for PUCN where no results have been published before), the column Diff lists the difference between the objective value of the computed solution and the previously best known published solutions (by August 1st, 2014) according to the DIMACS challenge (`http://dimacs11.cs.princeton.edu/instances/bounds20140801.txt`). In all other cases, the difference to the best primal solutions produced during the exact runs after one hour is given.

Table 48. STP PUCN instances remaining unsolved by B&C after one hour. Results computed through y-model-based local branching.

				BEST		AVG		STD		
Instance	$\|V\|$	$\|E\|$	$\|T\|$	UB	Time	UB	Time	UB	Time	Diff
cc10-2n	1024	5120	135	180	145.54	181.00	665.34	0.82	981.03	-3
cc11-2n	2048	11263	244	326	175.79	328.40	443.87	1.58	573.59	-5
cc12-2n	4096	24574	473	620	1263.20	621.70	1666.20	1.34	877.82	-6
cc3-10n	1000	13500	50	75	0.25	75.00	0.52	0.00	0.28	0
cc3-11n	1331	19965	61	92	0.36	92.00	0.83	0.00	0.52	0
cc3-12n	1728	28512	74	111	0.57	111.00	5.09	0.00	3.21	0
cc7-3n	2187	15308	222	289	812.00	290.90	888.70	1.10	616.65	-5
cc9-2n	512	2304	64	99	4.59	99.00	287.30	0.00	480.31	-1

Table 49. STP I640 instances remaining unsolved by B&C after one hour. Results computed through (x, y)-model-based local branching.

				BEST		AVG		STD		
Instance	$\|V\|$	$\|E\|$	$\|T\|$	UB	Time	UB	Time	UB	Time	Diff
i640-311	640	4135	160	35766	66.32	35778.60	1358.29	19.20	880.62	0
i640-312	640	4135	160	35770	455.41	35780.90	1899.64	26.38	1153.81	-1
i640-313	640	4135	160	35535	51.37	35535.00	597.67	0.00	511.78	0
i640-314	640	4135	160	35532	2981.68	35545.70	1623.32	7.78	973.20	-6
i640-315	640	4135	160	35720	634.72	35755.00	1115.30	14.28	928.91	-21
i640-321	640	204480	160	31100	3.99	31123.60	3.63	23.39	0.71	6
i640-322	640	204480	160	31078	4.82	31103.10	3.96	24.47	0.84	10
i640-323	640	204480	160	31081	3.95	31116.70	3.83	26.96	0.71	1
i640-324	640	204480	160	31095	4.06	31110.40	3.77	19.04	0.68	3
i640-325	640	204480	160	31084	2.71	31111.80	3.31	18.80	0.55	3
i640-341	640	40896	160	32086	2252.62	32114.50	1041.65	18.17	1099.36	44
i640-342	640	40896	160	32006	2009.31	32015.80	1333.93	3.52	1350.67	28
i640-343	640	40896	160	32040	180.36	32052.90	801.53	18.78	960.14	25
i640-344	640	40896	160	32051	768.36	32057.60	585.79	5.21	961.28	60
i640-345	640	40896	160	32020	107.07	32020.00	436.93	0.00	317.61	26

Table 50. STP PUC instances remaining unsolved by B&C after one hour. Results computed through CFT for instances with a set cover structure (bip*u/p, hc*u/p), and through (x, y)-model-based local branching for the rest.

				BEST		AVG		STD		
Instance	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	UB	Time	UB	Time	UB	Time	Diff
bip42p	1200	3982	200	24657	63.94	24667.60	903.38	14.18	1191.50	0
bip42u	1200	3982	200	236	47.68	236.00	714.40	0.00	631.56	0
bip52p	2200	7997	200	24554	787.88	24567.70	1169.34	12.88	1128.56	19
bip52u	2200	7997	200	233	1368.55	233.80	277.51	0.42	575.62	-1
bip62p	1200	10002	200	22906	3.47	22907.00	53.85	1.05	78.51	36
bip62u	1200	10002	200	219	6.30	219.00	12.12	0.00	4.83	-1
bipa2p	3300	18073	300	35355	540.54	35360.90	1303.05	4.38	867.45	-24
bipa2u	3300	18073	300	337	177.88	337.00	305.12	0.00	211.13	-4
cc10-2p	1024	5120	135	35298	289.17	35393.20	636.55	82.32	984.51	-81
cc10-2u	1024	5120	135	342	47.91	342.50	1137.75	0.53	1427.98	0
cc11-2p	2048	11263	244	63635	1227.09	64007.40	1200.23	189.87	726.29	-191
cc11-2u	2048	11263	244	614	1711.63	616.30	774.72	1.42	822.16	0
cc12-2p	4096	24574	473	124053	2265.43	124695.90	1455.49	383.05	880.76	2947
cc12-2u	4096	24574	473	1187	1587.62	1190.50	1768.84	3.27	787.39	8
cc3-10p	1000	13500	50	12782	3377.96	12820.30	2143.62	38.52	1377.73	-78
cc3-10u	1000	13500	50	125	19.69	125.00	211.44	0.00	362.89	0
cc3-11p	1331	19965	61	15594	1383.32	15638.30	1682.92	35.32	1242.23	-15
cc3-11u	1331	19965	61	153	13.16	153.00	148.57	0.00	149.05	0
cc3-12p	1728	28512	74	18837	2322.49	18905.50	1624.50	43.36	1182.90	-1
cc3-12u	1728	28512	74	185	29.07	185.00	150.37	0.00	163.39	-1
cc3-5p	125	750	13	3661	1.18	3661.00	12.61	0.00	21.88	0
cc5-3p	243	1215	27	7299	63.00	7299.00	1674.26	0.00	1350.22	0
cc6-3p	729	4368	76	20293	3292.58	20367.70	2153.67	40.45	894.91	-163
cc6-3u	729	4368	76	197	54.97	197.00	370.62	0.00	369.23	0
cc7-3p	2187	15308	222	57140	1330.80	57722.00	1734.91	297.56	1032.16	52
cc7-3u	2187	15308	222	550	1148.94	554.20	1346.71	2.35	972.56	-2
cc9-2p	512	2304	64	17202	891.76	17291.10	1052.64	40.31	1101.80	-94
cc9-2u	512	2304	64	167	664.86	167.20	1286.30	0.42	933.56	0
hc8p	256	1024	128	15322	1004.61	15345.12	1075.54	13.20	1002.10	0
hc8u	256	1024	128	148	0.04	148.00	0.17	0.00	0.12	0
hc9p	512	2304	256	30319	1873.63	30341.86	1906.07	16.90	872.74	61
hc9u	512	2304	256	292	0.29	292.00	0.52	0.00	0.20	0
hc10p	1024	5120	512	59981	256.73	60035.75	1324.04	34.71	918.08	-513
hc10u	1024	5120	512	575	11.33	575.00	115.98	0.00	143.77	-6
hc11p	2048	11264	1024	119404	3502.65	119532.00	2007.50	61.17	1279.12	-375
hc11u	2048	11264	1024	1145	666.33	1145.50	1481.97	0.53	1020.23	-9
hc12p	4096	24576	2048	236267	2764.17	236341.88	2309.54	61.04	656.21	-682
hc12u	4096	24576	2048	2261	2833.56	2262.89	2497.36	1.45	493.16	-14

Table 51. STP LIN instances remaining unsolved by B&C after one hour. Results computed through (x, y)-model-based local branching.

				BEST		AVG		STD		
Instance	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	UB	Time	UB	Time	UB	Time	Diff
lin24	7998	14734	16	15076	947.02	15076.00	1638.87	0.00	491.17	0
lin26	8013	14749	30	21782	216.96	21842.70	1806.49	35.34	1573.43	25
lin27	8017	14753	36	20693	143.36	20711.80	1337.22	11.26	1396.57	15
lin28	8062	14798	81	32622	3295.36	32666.30	657.99	17.61	1269.49	38
lin29	19083	35636	24	23765	90.57	23923.70	663.39	58.12	712.99	0
lin30	19091	35644	31	27752	264.14	27774.10	939.12	18.99	1065.90	68
lin31	19100	35653	40	31726	732.76	31749.10	1112.83	27.04	749.14	30
lin32	19112	35665	53	39937	274.26	40003.40	911.02	61.97	995.94	105
lin33	19177	35730	117	56215	3557.39	56274.10	2255.11	42.38	1537.64	154
lin34	38282	71521	34	45064	355.96	45155.80	844.17	45.07	914.03	46
lin35	38294	71533	45	50729	1885.32	50839.40	876.17	68.01	915.82	170
lin36	38307	71546	58	55794	592.60	55911.50	1364.44	69.79	1223.61	186
lin37	38418	71657	172	99744	679.93	99898.00	621.14	125.79	647.77	184

Table 52. PCSTP I640 instances unsolved by B&C after 1 hr. Results computed by (x, y)-model-based local branching.

				BEST		AVG		STD		
Instance	$\|V\|$	$\|E\|$	$\|T\|$	UB	Time	UB	Time	UB	Time	Diff
i640-311	640	4135	160	33503	47.70	33512.60	779.11	18.20	912.16	-85
i640-312	640	4135	160	32721	15.71	32731.90	1189.43	23.08	1217.38	0
i640-313	640	4135	160	32401	188.80	32402.80	1888.18	5.69	1314.10	0
i640-314	640	4135	160	32871	3607.58	32887.80	1573.59	14.58	1141.60	-96
i640-315	640	4135	160	32616	3044.16	32631.60	2120.19	10.22	1230.87	-72
i640-321	640	204480	160	28791	402.90	28805.00	368.22	12.08	229.29	-1
i640-322	640	204480	160	28469	1677.61	28483.00	520.95	10.15	467.06	-24
i640-323	640	204480	160	28160	273.64	28168.70	627.98	8.15	642.21	5
i640-324	640	204480	160	28746	402.34	28771.30	975.86	12.07	685.69	-11
i640-325	640	204480	160	28385	1743.53	28395.00	814.45	8.39	655.35	-19
i640-341	640	40896	160	29730	1363.31	29741.60	1777.39	14.66	764.77	33
i640-342	640	40896	160	29816	2914.08	29848.40	1729.33	24.22	1220.01	-28
i640-343	640	40896	160	30054	1756.09	30058.10	1039.93	4.33	954.59	-2
i640-344	640	40896	160	29926	2747.91	29943.60	2499.03	8.28	847.79	-51
i640-345	640	40896	160	30026	74.04	30046.30	965.82	18.09	1168.45	-5

Table 53. PCSTP PUCNU instances unsolved by B&C after 1 hr. Results computed by y-model-based local branching.

				BEST		AVG		STD		
Instance	$\|V\|$	$\|E\|$	$\|T\|$	UB	Time	UB	Time	UB	Time	Diff
bipa2nu	3300	18073	300	324	128.91	324.10	878.05	0.32	714.54	-1
cc10-2nu	1024	5120	135	168	60.93	169.50	654.03	0.85	1044.38	-4
cc11-2nu	2048	11263	244	304	168.00	307.60	375.00	1.43	826.78	-8
cc12-2nu	4096	24574	473	569	141.76	571.60	429.83	1.35	938.85	-14
cc3-10nu	1000	13500	50	61	4.28	61.00	61.53	0.00	62.49	0
cc3-11nu	1331	19965	61	79	6.81	79.00	335.97	0.00	346.63	-1
cc3-12nu	1728	28512	74	95	0.90	95.00	71.13	0.00	158.18	-1
cc7-3nu	2187	15308	222	271	3486.30	273.40	466.59	1.43	1066.66	-7

Table 54. PCSTP H and H2 instances remaining unsolved by B&C after one hour. Results have been computed through local branching using the y-model for uniform instances (hc*u(2)). For small non-uniform hypercubes (up to hc9p(2)), (x, y)-model-based local branching has been applied. For all larger hypercubes CFT has been applied (cf. Section 4.1) as in the STP case, and only the set of potential terminals are considered as terminals.

				BEST		AVG		STD		
Instance	$\|V\|$	$\|E\|$	$\|T\|$	UB	Time	UB	Time	UB	Time	Diff
hc10p	1024	5120	1024	59866	923.18	59965.20	1020.05	46.36	840.64	-567
hc10u	1024	5120	1024	559	1167.11	559.60	1484.19	0.89	1276.46	-2
hc11p	2048	11264	2048	119191	3600.29	119377.60	1850.72	96.35	1160.39	-2145
hc11u	2048	11264	2048	1116	506.03	1117.40	815.41	1.34	1288.41	-3
hc12u	4096	24576	4096	2221	229.83	2223.17	1259.94	2.14	1080.17	-5
hc12p	4096	24576	4096	240202	1966.87	240699.60	2182.22	352.97	789.42	-297
hc12p	4096	24576	4096	235860	2328.66	236098.20	2368.52	145.57	793.84	-4639
hc7p	128	448	128	7721	31.97	7721.90	1490.85	1.45	1337.39	-3
hc8u	256	1024	256	143	20.21	143.00	168.13	0.00	130.74	0
hc8p	256	1024	256	15213	101.22	15245.20	1852.15	18.81	1057.57	-65
hc9u	512	2304	512	283	303.28	283.00	1882.00	0.00	1502.92	0
hc9p	512	2304	512	30113	145.01	30147.80	862.96	26.01	1029.36	-239
hc10p2	1024	5120	1024	59930	2173.54	59966.30	1748.53	21.42	940.20	-954
hc10u2	1024	5120	1024	380	320.97	380.30	1355.36	0.48	1224.63	-3
hc11p2	2048	11264	2048	119236	328.77	119381.80	1094.65	89.67	673.23	-1898
hc11u2	2048	11264	2048	752	173.11	752.50	591.98	0.53	522.67	-8
hc12p2	4096	24576	4096	235687	3288.93	235974.70	2033.93	206.49	1108.63	-4496
hc12u2	4096	24576	4096	1493	358.26	1494.10	1295.30	0.99	1148.77	-22
hc8p2	256	1024	256	15261	2083.83	15269.60	1772.73	10.27	941.79	-35
hc9p2	512	2304	512	30242	30.15	30260.80	343.73	15.65	653.59	-281
hc9u2	512	2304	512	190	18.59	190.00	303.59	0.00	449.01	-1